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AUTHOR	Parfet, James A.
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ABSTRACT

This document is meant to be used as a teaching aid to help business teachers in Pennsylvania high schools prepare pupils to assume positions in business offices. Methods are suggested by which business mathematics may be presented to develop the greatest level of pupil achievement. The chapters outline business mathematics in the high school curriculum, teaching business mathematics, mastering the fundamental processes, fractions, decimals, percents, developing problem solving techniques, and evaluation. Appendixes include planning for individualized instruction, publications (books and free and inexpensive materials) and understanding negative numbers. (KP)

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BUSINESS MATHEMATICS

for

Business Education Departments

in

Pennsylvania's Public Schools

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Business Mathematics for Business Education Departments in Pennsylvania's Public Schools

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by James A. Parfet, Chairman
Business Education Department
Cumberland Valley High School
Mechanicsburg, Pennsylvania

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Milton J. Shapp, Governor

Department of Education
John C. Pittenger, Secretary

Office of Basic Education
Donald M. Carroll Jr., Commissioner
Harry K. Gerlach, Deputy Commissioner

Bureau of Vocational Education
John W. Struck, Director

Pennsylvania Department of Education
Box 911
Harrisburg, Pa. 17126

MESSAGE FROM THE SECRETARY OF EDUCATION

The preparation of publications designed to assist school administrators and teachers in improving classroom instruction is one of the functions of the Department of Education. Bulletin 279, *Business Mathematics,* is the latest in a series of business education curriculum guides—a series that started in 1956 with Bulletin 272, *General Business.* Other guides include Bulletin 273, *Bookkeeping;* Bulletin 274, *Office Practice;* Bulletin 275, *Typewriting;* Bulletin 276, *Data Processing;* Bulletin 277, *Shorthand;* and Bulletin 278, *Business English.*

In their evaluation of high school business graduates, businessmen frequently are critical of the mathematical background of these graduates. Because of its importance in the development of well-qualified office personnel, the quality of instruction in business mathematics needs to be improved.

An inexperienced teacher should find this publication useful in giving proper direction and emphasis in instruction. An experienced teacher should find it helpful in improving and vitalizing teaching procedures. Through the use of this guide, the quality of instruction in business mathematics should be improved. With business teachers striving for similar goals, business education will make a greater contribution to the educational objectives of Pennsylvania's high schools.

John C. Pittenger

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PREFACE

The past 15 years have brought about changes in the curriculum as well as in methodology of mathematics; business teachers need to be aware of implications these changes may have for business education. Bulletin 279, *Business Mathematics,* highlights some of these changes, offers reasons for them and discusses possible implications for business mathematics.

This publication has several purposes. First, it suggests methods by which business mathematics may be presented to develop the greatest level of pupil achievement. Second, it is a teaching aid which meets the needs of business teachers in Pennsylvania's high schools currently preparing pupils to assume positions in business offices. Third, it provides evidence of current practices in business mathematics as suggested by teachers and authors in this subject matter field.

As the content matter of Bulletin 279 is used and the suggestions and ideas are further developed, communicate your reactions to William H. Selden Jr., senior program specialist, Business Education, Department of Education, Box 911, Harrisburg, Pa. 17126, indicating recommendations for future revisions.

CHAPTER 1

BUSINESS MATHEMATICS IN THE HIGH SCHOOL CURRICULUM

It is no exaggeration to say that mathematical symbols are second only to the alphabet as an instrument of human progress.

—Stephen Leacock

Mathematics adds precision to communication through the use of carefully defined symbols and terminology. As a language, it uses ideograms, which are symbols for ideas, rather than phonograms, which are symbols for sounds. Ideograms are mental labor-saving devices, enabling persons to perform computations and to solve problems which would otherwise be difficult in written or spoken language. These symbols have the advantage of being understood by most educated people throughout the world, no matter in which language they converse. The equation $2 + 4 = 6$ means the same to a Norwegian or a Nigerian as it does to an American.

Responsible citizens need to understand these symbols fully, to work with them skillfully and to apply them intelligently to problems occurring in personal and business activities. Business mathematics is dedicated to this end. This chapter explores the definition of business mathematics, the relevancy and justification for its inclusion in the high school curriculum, the general objectives of this subject, suggested grade placement and a brief overview of mathematics education in transition.

Definition

Business mathematics is the reviewing and strengthening of the fundamental process of mathematics to develop competence. The basic concepts include addition, subtraction, multiplication, division, fractions, decimals, per cent and aliquot parts. Emphasis is placed on understanding the processes as well as accuracy and speed in computations.

After reviewing the fundamental processes, pupils' knowledge and skills are applied to problem solving in personal and business situations.

These situations include banking transactions; payroll procedures; personal and property insurance; investments; buying and selling merchandise; budgeting; local, state and federal taxation; weights and measurements; and collecting, reporting and interpreting business data. In addition, introductory units in bookeeping and accounting, formulas, the binary system, statistics and probability may be included.

For some office positions and for those pupils who wish to further their education in certain specialized fields, a knowledge of equations and finding the area and volume of geometric figures may be important.

Relevancy and Justification

Competence in business mathematics is needed by pupils. First, it will aid them in becoming effective consumers. Second, it will provide them with a background for success in business subjects requiring mathematical competence. Third, it will serve as a vocational tool in office positions requiring computational skills and problem solving techniques.

Numbers are used extensively in consumer situations; consequently, there is a vital need for understanding, computing and communicating in this symbolic language. Today's consumers find themselves inundated with numerical information in such situations as reading advertisements; watching television commercials and programs; purchasing merchandise on credit; paying local, state and federal taxes; borrowing money to buy a home; and making intelligent investments. In addition, the basic fundamentals of mathematics are used in diverse activities such as "do-it-yourself" building projects, homemaking activities, athletic contests and recreational activities. To completely divorce oneself from the world of numbers would be difficult.

Achievement in business mathematics is used by some instructors as an indication of a pupil's ability to successfully complete courses in bookkeeping or data processing. Specific units in business mathematics may be exploratory in nature, offering pupils guidance in their decision to pursue course work in these areas.

Bookkeeping and data processing require the use of computational skills and necessitate accuracy and neatness in recording numerical data. Furthermore, pupils will need to apply mathematical skill and knowledge in office practice when learning to use adding and calculating machines.

General Objectives

The objectives presented in the following list are considered the general goals of business mathematics. Specific behavioral objectives are discussed and examples are given in Appendix A, *Planning for Individualized Instruction*.

1. The pupil demonstrates—through classroom discussions—his appreciation of the importance of business mathematics and its relevancy to personal and vocational situations.
2. The pupil writes legibly and arranges numerical problems in acceptable form for computational purposes.
3. The pupil exhibits skill in estimating answers to business mathematics problems.
4. The pupil computes mental calculations accurately whenever practical.
5. The pupil applies shortcuts in computing whenever applicable.
6. The pupil performs with accuracy and facility the processes of addition, multiplication, subtraction and division.
7. The pupil performs, with facility, mathematical operations using whole numbers, fractions, mixed numbers, decimals and percents.
8. The pupil selects acceptable methods for solving problems in business mathematics.
9. The pupil solves accurately mathematical problems which occur in personal transactions and in business offices.
10. The pupil's class behavior demonstrates desirable personality traits, work habits, attitudes, trustworthiness and initiative—exemplifying the type of employe businessmen are seeking.

Grade Placement

As stated in the Department of Education's publication, *The Business Education Curriculum,* business mathematics generally is taught as a 10th-grade subject. The basic concept of this subject should be reviewed during the senior year in office practice or another business course. A review of business mathematics will help pupils sharpen their computational skills close to the time when they will need to use them on the job and will give them the confidence needed to complete successfully mathematical sections of employment tests.

Business mathematics fulfills one of the Pennsylvania Department of Education's high school graduation requirements of one planned course in mathematics during grades 10, 11 and 12.

Enrollment

Business mathematics, enrollment wise, ranks fifth among the business subjects offered in Pennsylvania's public secondary schools. During the

1970-71 school term, typewriting enrolled 178,654 pupils; bookkeeping, 48,124; general business, 46,717; shorthand, 40,464 and business mathematics, 38,245. Business subjects such as business English, business law, data processing, office practice, principles of selling and record-keeping had smaller enrollments than business mathematics.

Mathematics Education in Transition

Similar to other areas of knowledge, mathematics and mathematics education are continually expanding. Many persons who are not closely associated with the field of mathematics may have the opinion that all of the mathematics existing in the world today was created hundreds and thousands of years ago; however, Willoughby[1] reports that ". . . it has been reliably estimated that more mathematics and more significant mathematics, has been created in the 20th century than in all history prior to 1900."

Willoughby[2] briefly traces the history of mathematics education in the following paragraph:

> The history of mathematics education in this country has been marked by radical changes from its very beginning. In some places and at some time mathematics (including arithmetic) was not taught at all. On some occasions mathematics was taught because it was practical and necessary for life, and sometimes it was taught solely as a method to train the mind. Usually mathematics was taught in such a way as to encourage memorization rather than to bring about understanding.

Reasons for Transition

The new branches of mathematics and newly developed applications for old fields of mathematics, combined with other factors, have influenced course content and teaching methods in mathematics education in elementary and secondary schools. The following are some of the main reasons why drastic changes have been made in mathematics programs during the past 15 years:

1. Mathematics is an expanding field of knowledge.
2. New applications for mathematics have been developed.
3. Current learning theory stresses the importance of developing understanding through individual participation and discovery.

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[1] Stephen S. Willoughby, *Contemporary Teaching of Secondary School Mathematics*, (New York: John Wiley & Sons, Inc., 1967), p. 30.

[2] *Ibid.*, pp. 23-24.

4. Private foundations and the federal government have stimulated research and innovations through increased financial support.
5. The traditional mathematics programs lacked a desirable degree of success.

The "New Mathematics"

The new mathematics programs are characterized by new terminology and symbols; a reduction in emphasis of some topics; the elimination of other topics, including the consumer and personal-business applications; a transfer of some topics to lower grade levels and the introduction of entirely new topics. In addition, emphasis is placed on understanding why a process works, individualization of instruction, the discovery approach to learning, increased use of audio-visual aids, the structure of mathematics and the aesthetic qualities of mathematics.

Subject Matter Changes

Recently introduced mathematical topics in school curricula include numeration systems, sets, structures, vectors, matrices, probability, statistics, symbolic logic, non-euclidean geometry, transformations and computer programming. While many of these topics have originated during the past hundred years, several of them represent expansions of old fields of mathematics brought about through the discovery of new ideas and new applications. One example representative of the extension of an old field of knowledge is the binary numeration system. This system was invented in the late 1600s by the German mathematician and philosopher, Gottfried Wilhelm Leibniz. Until the 1940s, when the computer was developed, the binary numeration system was considered impractical.

Probably the greatest change in course content has been made in the intermediate and junior high schools. In the past, 7th and 8th grade mathematics courses were taught mainly for reviewing the basic fundamental processes with applications to consumer and personal-business situations. Currently, many programs in these grades consist of developing further insight into the operation of the fundamental processes and numeration systems, measurement, ratio (including per cent), geometric elements, graphs, formulas and elementary probability. Some schools have moved first-year algebra from the 9th grade to the 8th grade, while other schools have extended instruction to more advanced properties of numbers and more general treatment of graphs in the 8th grade.

The new topics in the mathematics program are meant to unify and clarify mathematics for pupils. The inclusion of the idea of *sets* in the elementary arithmetic program is a prime example. Sets are used to

describe a collection of objects, ideas or symbols; and they give meaning to the idea of numbers. In grades five through nine, sets are used to define basic operations as well as to define graphs as a set of points and to define probability as a set of events.

A basic aspect of modern mathematics programs is the stress on understanding the *why* of mathematical operations. The new mathematics approach demonstrates how a process works and teaches pupils basic laws—sometimes referred to as the 11 field properties[1]—that are used throughout mathematics. Previously, shortcuts such as *borrowing, moving the decimal point, transposing* or *canceling* were taught with little explanation into their meaning; whereas, topics taught in the new elementary programs are designed to develop a basic understanding of these processes.

Leaders in mathematics education emphasize that very little of traditional mathematics is being discarded. Number concepts, computational skills and knowledge of measurement are of fundamental importance for everyday applications; these topics are enriched through new insights provided by the new mathematics. Johnson and Rising[2] express their view as follows:

> What is valuable about the new school mathematics is not that it is new but rather that it offers an opportunity for students to learn mathematics more effectively, more pleasantly, and more meaningfully than has been possible before.

Refinement of Teaching Methods

Of significance to business mathematics teachers is the development and refinement of teaching methods. While teaching aids have been in use for hundreds of years, recent improvements increase the possibility of teachers doing a more effective job. Audio-visual aids, such as transparencies, filmstrips, 35 mm slides, records, tape recordings, television and plastic overlays in textbooks, are being used to enliven instruction.

The use of graphics in explaining and solving problems is becoming more common. The discovery approach in teaching problem solving, which is associated with modern mathematics, is currently being advocated. This method generally satisfies a pupil's desire to "do it myself!"

Emphasis is being placed on handling and manipulating objects to provide pupils with more effective learning experiences. Elementary teachers are using tongue depressors or popsicle sticks for counting and for developing understanding of the base ten numeration system. The abacus is used in some classrooms as a device for developing understand-

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[1] R. Robert Rosenberg and Harry Lewis, *Business Mathematics, A Modern Math Approach*, (New York: Gregg Division, McGraw-Hill Book Company, 1968), pp. 26-29.

[2] Donovan A. Johnson and Gerald R. Rising, *Guidelines for Teaching Mathematics*, (Belmont, California: Wadsworth Publishing Company, Inc., 1967), p. 23.

ing and competence in the fundamental processes. Adding and calculating machines are used in some high school and college mathematics classes to motivate, and to speed up, the learning of mathematical concepts and problem solving techniques.

Keeping Abreast of Changes

Business mathematics teachers need to keep abreast of changes in the mathematics curriculum and in methodology. Pupils enrolled in 10th grade business mathematics courses in all probability have had nine years of formal mathematics education. If the 10th grade business mathematics teacher is oblivious to a pupil's previously acquired knowledge concerning procedures of operation and basic terminology, effective communication may be impeded. Misunderstandings or misinterpretations might cause confusion resulting in low levels of achievement on the part of the pupils and in frustration on the part of the teacher.

Business mathematics teachers may keep up to date with changes in mathematics education in one or more of the following ways:

1. Suggest and encourage interdepartmental meetings between business teachers and elementary, intermediate and secondary mathematics teachers.
2. Request, organize and participate in business mathematics workshops sponsored by business teacher organizations such as the Pennsylvania Business Education Association and the National Business Education Association.
3. Secure and read copies of arithmetic and mathematics textbooks used in elementary and intermediate grades.
4. Enroll in college and university seminars in modern mathematics.
5. Browse through publications of the National Council of Teachers of Mathematics, such as *The Arithmetic Teacher* and *The Mathematics Teacher,* for articles which may be pertinent to business education. Periodically, the NCTM publishes booklets which may be of interest and use to teachers of business mathematics.

Just as businessmen are quite concerned about *research and development* in their respective companies, business teachers need to be interested in research and development in education. Using the five resources mentioned, teachers should search continually for new ideas which might be developed into effective teaching techniques.

CHAPTER 2

TEACHING BUSINESS MATHEMATICS

. . . teaching mathematics is a complex task . . . it involves not only mathematics, methods, and materials, but it involves human beings, each of whom has physical, intellectual, and emotional reactions.

—Donovan A. Johnson and Gerald R. Rising

The teaching of business mathematics should be a challenging and an exciting adventure. The challenge lies in stimulating pupils into wanting to learn the subject matter to be presented, and the excitement occurs in the procedures and techniques used to teach it.

Pupils

All pupils should be urged to keep readily available the materials they will need for business mathematics such as textbooks, pencils, erasers, papers and possibly small plastic rulers. The practice of borrowing materials indicates carelessness, wastes time and causes disturbances in the class. The teacher should keep a limited amount of materials on hand to provide for emergencies. When necessary, a pupil should borrow materials from the teacher rather than interrupt a classmate.

Pupils need to realize that they have the responsibility to ask for the teacher's assistance during assigned class periods, before school, after school or during a study period. Pupils need to be aware of their strengths and weaknesses in mathematics through self-evaluation, and they should be encouraged to accept responsibility for attaining specific and realistic goals.

For instruction to be effective, the teacher should be aware of the attitudes, interests and abilities of the pupils. Information concerning pupils may be attained in the high school office from their cumulative records. In addition to grades in previous mathematics courses, ability in mathematics may be indicated by scores on previously administered achievement or diagnostic examinations.

This background information can serve as the basis for planning initial instruction for a particular group of pupils. Initial instruction in

business mathematics should be at a level with which pupils are familiar and can demonstrate competence. After pupils have achieved success with familiar material, the time has come to move toward new areas of learning.

The Teacher

Teachers should periodically evaluate their attitude toward their work and their pupils. Questions such as the following need to be asked and answered by each teacher: "Do I enjoy working with young people?" "Do I try to understand the reasons for pupils' attitudes and actions?" "Am I excited about my subject, and am I eager to convey my enthusiasm and my knowledge?" A wholesome classroom atmosphere, which stimulates learning and avoids the frustration of repeated failure, needs to be established. Some class time should be devoted toward developing appreciation for business mathematics and discussing the necessity for attaining competence.

Teachers can encourage pupils to participate in class activities by making eye contact with individuals to whom they are speaking, by providing an atmosphere where questions are welcome, by asking pupils questions which they are apt to answer and by instilling in them the fact that errors are inevitable in skill development. Discussion can be stimulated by asking questions which require more than "yes" or "no" answers.

The teacher has the responsibility of coordinating subject matter, instructional materials, classroom procedures, teaching techniques and evaluation instruments in the most efficient manner possible for the purpose of stimulating pupils toward the attainment of specific behavioral objectives as found in Appendix A. This implies that each lesson be planned systematically, using the most effective resources and the most logical teaching techniques at the appropriate time and for a particular group of pupils.

For self-evaluation purposes, business mathematics teachers can study and reflect upon the following criteria. The business mathematics teacher is one who (1) emanates enthusiasm for business mathematics as an instrument for pupil development; (2) is cognizant of the motivations and limitations of individuals and has the capability of relating to them; (3) demonstrates expertness in the skills and knowledge encompassing the field of business mathematics; (4) has experience in, and a thorough understanding of, office work and procedures which relate to the mathematical skills and problem solving techniques to be taught and (5) is knowledgeable of appropriate teaching techniques and devices to stimulate pupils in learning business mathematics.

Classroom Management

In the beginning of the school year definite classroom routines should be established and pupils advised of what is expected of them and what type of assistance they may expect from their teacher. Efficient classroom management is essential in the teaching-learning process. Careful organization of classroom activities can aid in preventing discipline problems and in providing pupils with a sense of security and direction.

Classroom tasks, such as taking attendance, making announcements, collecting homework, distributing paper, etc., should follow established patterns. Pupils need to share in performing these tasks to the degree possible. Sometimes pupils of lower scholastic ability gain self-confidence and develop improved attitudes toward a subject when they actively participate in routine classroom activities. Topics which might be discussed with pupils at the beginning of the year include the value of business mathematics, course content, pupils' classroom responsibilities, homework assignments and testing and grading procedures.

Preview

When new work is presented with little or no idea of the relationships of the individual details to the structure of the unit as a whole, some pupils lose interest and fail to comprehend the relevance of the details. Pupils might then be given a preview of a unit or topic in its entirety. The purpose of the preview is to give pupils a perspective of the major ideas and principles in the unit and their relationships to the unit as a whole as well as to previous topics. A preview should give meaning to the material, offer stimulation for studying the unit and provide for understanding and insight into the steps necessary to achieve mastery. Previews can be given in the form of brief, well-organized talks together with visual illustrations designed to arouse pupils' curiosity and interest.

Introducing New Concepts

The following procedure, using the chalkboard or overhead projector, for introducing a computational technique or a problem solving method is suggested:

1. The teacher works a sample problem on the chalkboard or on a transparency.
2. A similar sample problem is worked by the teacher at the chalkboard or on a transparency while pupils work the same problem at their desks.
3. A third problem, similar to the first two, is presented for pupils to work independently at their desks while the teacher observes as many pupils as possible.

4. If pupils encounter difficulty with the third problem, the teacher should work it with them and answer any questions that may arise.

5. When the teacher is satisfied that the pupils have gained insight in working a given type of problem, he or she then assigns a set of similar problems for classwork and/or homework.

When a new type of problem is introduced, use numerical data relevant to the class whenever possible. For example, in a lesson on finding averages, pupils might be interested in finding the average height of the girls and boys in the class. In a lesson on percentages, pupils might be interested in learning what percentage of their classmates hold part-time jobs or what percentage of pupils live in a certain municipality (if the school district is comprised of more than one township or borough). Innumerable data can be collected from a class or from the school's student body for use as examples in teaching problem solving.

Directed Study

Insofar as possible, the major portion of the class period should be devoted to pupils working problems at their desks. Directed study may reveal shortcomings in an instructor's presentations and barriers in pupils' thinking. Observing pupils successfully solving exercises and problems and helping individuals overcome learning barriers is one of the major responsibilities of the teacher.

When the directed study period begins, the teacher should make a quick check of the pupils at work and observe which ones seem to be having difficulty. If the teacher's observation reveals that a considerable number of pupils need assistance, he or she should stop the directed study session and reteach those principles which seem to be causing difficulty. This will not be a frequent occurrence if the teacher consistently evaluates the instructions by questioning a sufficient number of pupils while presenting a process or developing a concept.

If no additional instruction is deemed necessary, the teacher may give assistance to either individual or small groups of pupils. Pupils are expected to keep working and to continue in their efforts until the teacher works his or her way around the classroom. The teacher ought to avoid spending too much time with an individual pupil. If a learner seems to be having unusual difficulty, the teacher should schedule an individual session during a free period or before or after school when undivided attention can be given to the pupil.

Providing for Individual Differences. There are significant areas which have a bearing on pupils' learning rates. Pupils differ mentally, physically, emotionally and socially. They have varying interests, atti-

tudes and appreciations; and pupils differ in ability to read, to observe, to listen and to communicate. Their attention spans and retention capabilities vary considerably as well as their study habits, self-discipline and creativity. Young people come from different home environments and have had varying educational experiences in different schools and from various teachers. For these reasons, it is sometimes difficult to understand and to be understood by all individuals at all times.

To provide adequately for individual differences, teachers need to (1) vary course content to the needs of individuals; (2) present instruction in language that pupils can understand; (3) allow for varying rates of learning; (4) make assignments according to individual needs whenever possible; (5) provide instructional materials in various levels of difficulty; (6) use a variety of instructional techniques and (7) place emphasis on pupil participation.

Teachers interested in developing a unit or an entire business mathematics course on an individualized instructional basis should refer to Appendix A, which provides a sequence of steps to follow in preparing individualized instructional units.

Grouping. Teachers on the secondary level usually work with an entire class or with an individual pupil; however, business mathematics teachers may find it advantageous to group pupils, similar to the procedure used in elementary classrooms. One of the detriments to grouping on the high school level is that class control may be more difficult to maintain. When working in smaller groups, pupil discussions have a tendency to deviate from the lesson and some individuals in one group may divert their attention to another group.

Properly planned and conducted, grouping of pupils—according to abilities and past achievement records—can be worthwhile. One procedure that has worked successfully is the use of taped lessons, guide sheets and worksheets. In one group, the pupils wear earphones connected to a tape recorder. They listen and respond to a taped voice which tells them what to do on the worksheet. Due to the restriction of sound afforded by earphones, pupils' attention is held and they are oblivious to their classmates' activities.

A pupil-leader could work with a second group while the teacher instructs a third group, or two pupil-leaders could conduct groups while the teacher shares his or her time with individuals in all three groups. The preparation of job instruction sheets for use within the groups will help insure that pupils follow prescribed learning patterns.

The key to success in grouping is adequate planning and preparation of materials. The ideal situation would be for a teacher to use the summer months to plan and prepare materials. If this is not possible, each year the teacher might choose a unit of instruction and prepare materials throughout the school year for that topic in preparation for the ensuing year.

Assignments

Homework assignments can be varied by providing problems to maintain and improve skills and develop new concepts. The daily exercises for developing skills should be based on a planned sequence directed toward a given objective. Methods for solving review problems should be discussed and pupils should be aware of sample problems for reference. A directed study session as previously suggested should be an integral part of each class period. This procedure allows time for the teacher to observe pupils and to help them overcome any possible area of difficulty, and pupils are given a start on their homework with the assumption that they will be more inclined to complete the assignment. Some business mathematics teachers suggest weekly assignments to allow pupils the opportunity of planning their homework.

Suggestions for Independent Study

A teacher cannot assume that pupils have developed good study habits; and, when necessary, needs to teach them effective study techniques. Teachers may wish to provide pupils with a list of suggestions for independent study. Figure 1 is a suggested handout which may be used as a basis for a discussion on the importance of proper work habits.

Reviewing Homework

Some teachers have developed an effective routine for reviewing homework. Pupils who have had difficulty with certain problems in a homework assignment place the numbers of the problems on the chalkboard as they enter the classroom. Members of the class who have solved the problems in question place their solutions on the chalkboard for their classmates to follow, to check and to question. After the solutions to the problems have been placed on the chalkboard, a discussion concludes this activity. This procedure saves class time and gives the teacher an opportunity to take attendance or perform other related duties. In addition, pupils are provided with an opportunity of sharing in the learning process.

Another routine is to place a numbered list of the homework problems on the chalkboard; and, as pupils enter the room, they place a check mark after the numbers of the problems that were troublesome. When the teacher is ready to begin instruction, he or she can quickly ascertain which problems presented the most difficulty for the class by checking the list on the chalkboard and can spend the majority of the homework review time on those problems.

For some assignments, teachers may wish to prepare duplicated copies of solutions to assigned problems. As pupils enter the classroom, they may pick up a copy of the solutions and immediately begin to check

BUSINESS MATHEMATICS

STUDY GUIDE

1. Make certain you understand the assignment.
2. Write the assignment in your notebook or on a piece of paper.
3. Read carefully the explanations and instructions for solving the exercises and problems.
4. Use the index and reference guides in your textbook as sources of information on the definition of terms and symbols and for explanations of processes.
5. Ask questions when in doubt about a procedure.
6. Write neatly, keeping figures aligned and labeled; remember that every symbol and figure has a definite meaning.
7. Develop the checking habit to be sure that numbers have been copied correctly.
8. Use your textbook advantageously by referring to illustrations and to sample solutions—following them step by step.
9. Organize the assigned problems in the same manner.
10. Draw diagrams or graphs whenever necessary to help you see the relationships between numbers being used.
11. Develop the habit of estimating answers to computations before working them to be aware of the reasonableness of your answers.
12. Prove each computation by working it in a different way.

By participating and doing your share, you may discover that business mathematics can be an enjoyable and a rewarding experience!

Figure 1. Study guide

their homework. This technique provides pupils with a means of self-evaluation and saves valuable class time. When instruction is individualized and pupils receive differentiated assignments, this method is especially appropriate.

Chalkboard

The chalkboard is one of the more commonly used teaching aids in the classroom. It can be used advantageously by both teacher and pupils. Illustrations drawn on the chalkboard help pupils visualize the process being presented. Teachers need to set an example by writing neatly, by organizing work systematically and by labeling figures adequately. Excuses by the teacher such as "I'm writing this hurriedly," or "I'm not labeling figures to save time," are not necessarily valid.

When writing on a chalkboard, the teacher ought to turn his head toward the class every few seconds to be sure of pupil attention. Also, when explaining a problem on the board, he should face the group when talking to them. By so doing, a teacher avoids turning his back to the class to the degree possible.

Weaknesses in computation and problem solving may be determined by observing pupils' performances at the chalkboard. As pupils are called upon to work at the board, the instructor ought to stand in the back of the classroom. By doing this the teacher can see if those pupils at their desks are working on the correct exercises and if the pupils at the chalkboard are writing legibly.

Colored Chalk

Some teachers use colored chalk for identifying specific parts of a problem and for creating interest in the visual presentation. The teacher can use white chalk to show the facts and figures given within the problem, and then use yellow chalk to show the solution of the problem. A teacher who uses this method reports that, "This technique seems to work quite well with all types of pupils as well as with all types of problems." Another example would be to use red chalk to estimate answers, white chalk to work problems and yellow chalk to prove the work. When illustrating fractional parts of an amount or quanity by drawing "pies" on the chalkboard, one color of chalk could be used to section the pie into thirds and another color to section it into twelfths. One disadvantage of using some colored chalk is the difficulty in erasing. Chalkboards should be washed daily, especially when colored chalk is used.

Other Devices

For chalkboard use, teachers should have other devices available to facilitate the placing of illustrations on the board. These include a

heavy duty yardstick with a knob or handle, a compass and chalk templates. Chalk templates can be made by puncturing heavy paper at specific places to aid in drawing illustrations on the chalkboard.

Overhead Projector

The overhead projector can no longer be considered a "new" teaching aid. It has been in use for many years and schools are purchasing them in increasing quantities. Many recently constructed schools have installed projectors and screens in each classroom as part of the basic equipment.

Advantages

The main advantages of the overhead projector are: (1) the easy, flick-of-a-switch operation permits the teacher to turn the machine on and off throughout the presentation, thereby controlling the discussion and obtaining the pupils' attention; (2) the classroom does not have to be darkened while projecting illustrations, enabling pupils to take notes during the presentation; (3) illustrations can be prepared in advance and used innumerable times; (4) through the use of overlays, dynamic effects can be projected, adding interest and providing an organized, progressive development of a concept or process; (5) the overhead projector allows the teacher to face his pupils as he writes on, points to or adjusts a transparency and (6) the horizontal writing platform provides a convenient working surface.

Considerations

When subjected to continuous overhead projections for long periods, some individuals feel a visual strain. This is critical when pupils are exposed to projected instructional media in several classes during the day. To avoid visual strain the overhead projector should be used discriminately and the transparencies prepared carefully. If transparencies are poorly prepared and the projected images are too light or indiscernable, they will be unpleasant to look at and cause pupils to lose interest.

Teachers should be sure that the projected images are in focus and aware of the *keystone effect*—a distortion in which the top of the projected picture is wider than the bottom. If a distortion is minor, it may be ignored; however, if it is bothersome and throws the image out of focus, it needs to be corrected. The keystone effect may be rectified by tilting the top of the screen forward toward the projector. Tiltable screens are available and they minimize the keystone effect.

Preparing Transparencies

An instrument commonly used to write on transparencies is a grease pencil. This instrument is useful if the transparency is not to be kept

permanently as the grease markings can be wiped away and the acetate reused. For permanent markings, special pens and inks may be used.

The quickest, and possibly the best, method is to use a copying machine that will reproduce printed, typed and hand drawn material onto transparency film for overhead projection. Transparencies also can be prepared on a typewriter. A carbon ribbon machine will produce the neatest and most legible copy. To avoid smearing the typewritten image on the acetate, a second acetate sheet can be placed directly on top of the one on which the typewritten image appears. These two acetate sheets can then be fastened together with transparent tape. When using a transparency prepared on a standard typewriter, the distance from the overhead projector to the screen should be increased to provide larger print on the screen.

The fluid (spirit or chemical process) duplicator can be used to prepare transparencies. This method is especially convenient if the teacher plans to duplicate copies of certain material and also wishes to have a transparency of the same material for instructional purposes. After the fluid master has been attached to the duplicating machine, special commercially prepared acetate with the frosted side up is fed through the duplicator; then, the teacher may complete the run on duplicator paper for class distribution.

Lettering. Pressure-sensitive and transfer lettering, numerals, circles and arrows may be purchased for producing permanent transparencies. With pressure-sensitive materials, the backing is removed and the item is pressed on the transparency. With transfer lettering, the teacher places the translucent sheet of paper containing letters of the alphabet and rubs over the selected letter with an instrument such as a ballpoint pen. When the translucent sheet is removed, the letter remains on the transparency. Plastic templates also may be used as guides in preparing lettering and designs.

Color. Attention holding colors can be added to transparencies by purchasing transparent sheets of adhesive material in various colors. The material is cut to the size desired, the protective backing is removed, and the adhesive side is applied to the front of the transparency. Colored pressure-sensitive tapes are available and may be purchased in several widths. Felt tip marking pens also may be used to add color to illustrations on transparencies.

Overlays. Overlays are a series of transparencies used to illustrate the sequence of steps in problem solving or in a mathematical process. An original diagram should be prepared before attempting to make the overlays. The pieces of an overlay match to produce the desired illustration when the sequence is completed. Various colors can be used to identify the steps in a sequence. When using overlays, each step of

a process may be discussed as it occurs. A mimeoscope can be beneficial in the preparation of overlays and transparencies.

> Of the many aids to effective communication that have been offered to teachers, and especially to teachers of mathematics, the overhead projector seems to be one aid that may radically improve the teaching of mathematics.[1]

This statement implies that this visual method be employed effectively. A teaching aid is a help, but the teacher's planning and judicious use of the aid makes it helpful.

Community Resources

Occasional complaints of business mathematics teachers are, "My pupils don't like business math!" or "I can't seem to stimulate my pupils into wanting to learn business math!" To overcome lethargic attitudes such as these statements imply, teachers need to use vivid experiences which add realism to their courses. Resources which can be used to enrich business mathematics instruction include field trips, slide presentations, speakers, newspapers and other supplementary materials. Also, a class research project, using data gathered from the community, can be conducted at least once during the year.

Field Trips

A field trip to a local business organization or governmental agency is often overlooked as an instructional tool in business mathematics. Opportunities for trips vary greatly, depending upon the location of the school. Suggestions for visitations include governmental agencies, utility companies, transportation facilities, banks and other financial institutions, insurance companies, retail stores and manufacturing firms.

Pupils may observe the computation of wages, costs, depreciation, overhead, interest, dividends, commissions, department budgets, markups, etc. Most business enterprises collect data, prepare statements, gather statistical information and determine probability. In addition, pupils may observe adding machines, calculators and computers in operation and may see a variety of business forms being used to facilitate the compilation, computation and communication of numerical data.

The objectives of a field trip are to (1) improve pupils' attitudes concerning mathematics and its usefulness and importance in society; (2) relate school work to reality in the community; (3) provide interest and stimulation in a unit of study and (4) help pupils recognize relationships between business mathematics and knowledge learned in other subjects

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[1] Stephen Krulik and Irwin Kaufman, *How to Use the Overhead Projector in Mathematics Education*, (Washington, D.C.: National Council of Teachers of Mathematics, 1966), p. 1.

in the school's curriculum. For example, pupils may learn about taxation in general business; then, in a field trip to a county courthouse they may view the taxation process in action and observe how mathematics is used to arrive at, compute and collect taxes.

Adequate planning will help ensure the success of a field trip. This type of activity is costly due to expenses incurred by the school district for transportation and by the business for use of personnel. In addition, the time spent by pupils, a teacher and businessmen should be productive in terms of value received from the visitation. The following items should be considered along with administrative details when planning a trip.

1. Prepare a list of mathematical operations to be observed.
2. Visit the business or agency prior to the trip to meet those responsible for conducting the trip to discuss objectives and possible observations, to check available facilities and to gather information for preparing pupils for the trip.
3. Prepare pupils for the trip by discussing terminology and reviewing mathematics applicable to the procedures to be observed. This may be accomplished by having pupils follow a flow chart or study a guide sheet.
4. Discuss observations with pupils as soon as possible after returning from the visitation.

Slide Presentations

To take more than one or two field trips with any one class during a school year is not practical. Therefore, teachers can produce their own slide presentations, which can be used as an adjunct to visits outside the classroom. Teachers can visit a business or organization to take slide photographs of mathematical operations in the local community. A flow chart and a guide sheet may be prepared to provide data relating to the visual trip.

Slide presentations of this type have some advantages over a field trip. Pupils' attentions can be focused on the operation while the teacher explains the process. This type of presentation does not require taking a large group of pupils out of school, thereby eliminating the need for additional chaperons and the expense of transportation. Also, pupils will not be absent from other classes because of a field trip.

The slide presentation can be updated each year, and additional presentations can be produced periodically. Slide presentations are not intended to take the place of well-planned field trips; they are presented here as another means of varying presentations to help stimulate pupils.

Speakers

Many businessmen and women are willing to cooperate with teachers and enjoy speaking to young people concerning their fields of employment. Inviting one or two businessmen or women to speak to the class each year can be another means of stimulating pupils. A person representing a firm or organization outside the school environment sometimes can influence and make an impression on some pupils whom the instructor may otherwise be unable to arouse. Organizations of businessmen and women, such as the Administrative Management Society, willingly provide teachers with lists of prospective speakers on various topics pertaining to their particular fields and interests.

Newspapers

An additional community resource which can be used to enrich business mathematics instruction and to stimulate pupils is the daily newspaper. Through the newspaper, pupils can be taught to relate whole numbers, decimals, fractions, percents and financial problems to people, to things and to events. In addition to learning business mathematics an important result of using the newspaper in the classroom is that pupils are developing the habit of reading the newspaper and analyzing its content—a skill which will serve them the rest of their lives and help them to become better citizens.

The newspaper can be used to introduce pupils to the subject of business mathematics as well as to the make up of a newspaper. By using a bulletin board display or posters depicting the various sections of a newspaper, a teacher can illustrate how numerical data and news which influence these data appear in bannerlines, headlines, news stories, pictures, charts, graphs, feature stories, financial reports, editorials, sports and advertisements.

When introducing pupils to specific topics or units of instruction, the teacher may assign pupils to clip articles and advertisements from the newspaper in which pertinent mathematical data appear. In this manner the importance and relevance of the topic to be developed can be discussed before actual exercises are assigned. After the discussion, exercises involving that particular mathematical topic may be assigned from the textbook. After pupils reach a degree of competence in performing the computations involved, their knowledge and skill may be applied to problem situations, some of which may be found in sections of the daily newspaper.

Bannerline and Headlines. Figures often appear in the bannerline or headlines throughout the newspaper and pupils need to be taught how to read and interpret these figures. For instance, consider the headline, "School Subsidy Bill Will Cost $105.8 Million More for '72." Pupils

can be questioned as to how that amount would appear in figures and why editors write $105.8 million instead of $105,800,000. The bannerline, "2.3 pct. flat levy on income anticipated," could be developed into a problem situation by assigning pupils to compute the state income tax on various weekly wages some of their classmates may earn on their part-time jobs. The bannerline and headlines also may contain news about situations which may influence business statistics. For instance, pupils might be asked to react to the impact the following headlines have had on the prices of particular stock issues: "Nixon removes excise tax of 7 per cent from autos," or "Senator says railroads to go out of business."

News Stories. News stories contain figures that need to be read and interpreted. Many of these stories contain figures expressing population increases or decreases, crime rates and governmental budgets. These stories can be used to develop problem solving exercises.

Pictures, Charts and Graphs. Many photographs are printed which have a bearing on financial data. When studying property insurance, pictures of local fires and accidents can be used to stimulate discussion on the types of property insurance, the necessity for having insurance and the cost of insurance (possibly including a study of probability in its elementary aspects). Newspapers occasionally print charts and graphs which can be used advantageously in this course. When teaching personal and property taxes, a more realistic and beneficial practice for pupils would be to use actual tax rates established in their own community. As these charts and graphs appear in the newspaper, they should be clipped, saved and used by the class when the unit on taxation is taught.

Financial News and Reports. The newspaper is the natural resource to use when instructing pupils in the study of stocks and bonds. This source is up to date and the changes which occur daily in the stock and bond market quotations are sources of mathematical problems. When using the newspaper to study stock and bond quotations, the teacher can use the textbook as a reference source which provides explanations for the figures listed along side the names of the stocks and bonds.

Sports. The major league baseball standings are an excellent source of percentage exercises. Classes might be asked to refigure the league standings both on the assumption that the teams win and on the assumption that the teams lose their next game. Pupils, especially boys, would be more interested in calculating these statistics than they would in some of the exercises in the textbook. This is not to imply, however, that the textbook is not a valuable instructional tool.

Advertisements. Newspaper display advertising can be the source of many types of exercises and problems. A tire ad indicating "25% Off on All Roadmark Polyester Cord Tires"—listing the regular price and the sales price in eight sizes—can be assigned to pupils for checking the accuracy of the sales price as listed in the newspaper. Commodities such as liquid dish washing soap and toiletries are used as leading items to entice buyers into stores. When the same product is advertised by different retailers in various sizes and at different prices, pupils can be assigned the task of comparing the prices to determine the best buy.

The success of the newspaper as a teaching aid in the classroom will depend to a great extent on how enthusiastic and creative the teacher may be.

Supplementary Materials

Free and inexpensive instructional materials are available from businesses, associations and governmental organizations. Some of these materials are appropriate for remedial work, others to supplement the textbook and still others for enrichment purposes. In addition, posters and pictures may be secured for display on bulletin boards.

Booklets such as *Policies for Protection* can be used to supplement the textbook in the study of life insurance. *Chances Are* (an introduction to probability using programmed instruction) is a publication which can be used as an enrichment exercise for those pupils who may have been excused from a unit on fundamentals because of their competence in that area of mathematics. The publication, *You and the Investment World,* offers additional information on stocks and bonds from that contained in business mathematics textbooks and, in addition, provides the teacher with ideas for projects and bulletin boards. The *booklet, From Og . . . to Googol,* is an interesting publication which could be used as the basis for an oral report if a unit on the history of numbers and the development of numeration is a part of the course of study. Sources of the above publications are listed in Appendix B. *Publications.*

The gathering and use of supplementary materials in the classroom might follow this sequence:

1. Organize a resource file according to units and topics.
2. Discover available materials through reading professional journals and attending professional meetings.
3. Secure pertinent materials.
4. Analyze their appropriateness.
5. Select and retain useful data and ideas.
6. Place materials in the resource file for future reference and use.

7. Use selected materials in the instructional program.
8. Evaluate their effectiveness.

Research Projects

Business pupils need to be exposed to practical situations whereby they are required to gather, organize, analyze, compute and present numerical data in a meaningful way. This type of activity might be the logical way to conclude the course, putting many of the skills and problem solving methods to use in a single research project.

A suggested class project would be to have pupils compare the differences in cost and travel time pertaining to various modes of transportation—airplane, automobile, bus and train. This activity would involve the following steps:

1. Decide on several places the class would like to visit.
2. Secure the cost and travel time for various modes of transportation to the chosen destinations.
3. Distribute the collected data in duplicated form to all class members.
4. Decide, as a class, how the data should be organized and what mathematical calculations should be made.
5. Discuss ways of presenting the data for dissemination—written, chart or graphic form.
6. Assign a different method of organizing and presenting the data to each row of pupils.

The project could culminate in a bulletin board project where maps, pictures of places to be visited, timetables, travel advertisements and copies of some of the outstanding presentations developed by individual pupils could be displayed in an appealing manner.

Another research project might involve the comparing of costs of advertising by newspaper, radio, television or billboard. Other research projects could develop from the vast amount of numerical data available within a high school. Pupils may have some suggestions concerning information they would like to know about their own school and student body; for instance, how much does it cost the local school district to educate one pupil?

Conclusion

While this chapter contains many suggestions for procedures, audio-visual aids and resources, teachers have the responsibility to choose and to adapt these aspects of instruction to their unique teaching situations.

CHAPTER 3

MASTERING THE FUNDAMENTAL PROCESSES

The best preparation for tomorrow's work is to do your work as well as you can today.

—Elbert Hubbard

Competence in business mathematics requires the mastery of the fundamental processes—addition, multiplication, subtraction and division. Not all pupils have mastered computational skills during their previous years of education, and many pupils need to have these skills sharpened. Teachers, therefore, should give adequate consideration to this important phase of business mathematics before embarking on problem solving exercises. Psychology of learning suggests an emphasis on stimulation, understanding, particpation and reinforcement as the steps to computational skill mastery.

The scope of this chapter is twofold. First, techniques are provided for teachers to use in guiding students toward an understanding of the fundamental processes. Second, specific suggestions are provided for guiding pupils toward the mastery of these processes.

Addition

Of the four fundamental calculations of arithmetic, addition is used in business offices and in personal lives more often than any of the others. Some pupils might be heard to retort: "I'm not going to waste my time adding columns of figures; everyone uses adding machines today!" Rosenberg and Lewis[1] point out the following: "Saying that arithmetic skills are useless because machines now do all the work is like saying that one doesn't need to learn to write because we have typewriters." How many persons use an adding machine to total the score in a card game or to record the number of points individual members of a basketball team

[1] R. Robert Rosenberg and Harry Lewis, *Business Mathematics, A Modern Math Approach*, (New York: Gregg Division, McGraw-Hill Book Company, 1968), p. 20.

score? Many employes in restaurants, stores and business offices can be observed performing calculations without the use of adding machines.

Accuracy and speed can be developed through awareness of and conscientious practice in recalling the 45 addition facts avoiding verbalization when combining digits, grouping digits to form combinations of 10 and using subtraction advantageously. Futhermore, the proving of addition can be done in several interesting ways.

Addition Facts

Addition facts are sets of two single addends equaling a sum. While most pupils have learned the addition facts in elementary school, some may need to practice the more difficult combinations to sharpen this skill. This practice should be done by thinking the sum of the combinations rather than by counting. Forty-five addition facts are presented in Figure 2. By having pupils who need this type of practice recall the sum of the combinations in the order in which they are presented $(4 + 7 = 11)$ and in reverse order $(7 + 4 = 11)$, they will be practicing all the combinations. To vary their practice, pupils may work through the table horizontally, vertically, diagonally left and diagonally right.

Avoiding Verbalization

Some pupils have the habit of verbalizing the sum of two digits by thinking to themselves, "five plus nine equals fourteen." They should be urged to discontinue this practice. Pupils may gain insight into this shortcoming if the teacher uses an analogy from reading. When pupils see the word *desk*, they usually don't verbalize *d - e - s - k* spells desk; therefore, when the combination 5 + 9 is observed, they should *think* 14.

Grouping

Although the grouping of digits to form combinations of 10 is covered in most business mathematics textbooks, supplemental practice materials may be required for some pupils. Duplicated supplementary work sheets or printed workbooks, such as the *Mathematics Skill Builder,*[1] which contain exercises of this type, should be available and assigned to pupils who need this additional practice.

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[1] Milton Briggs, *Mathematics Skill Builder,* (Cincinnati: South-Western Publishing Company, 1969) pp. 9-10.

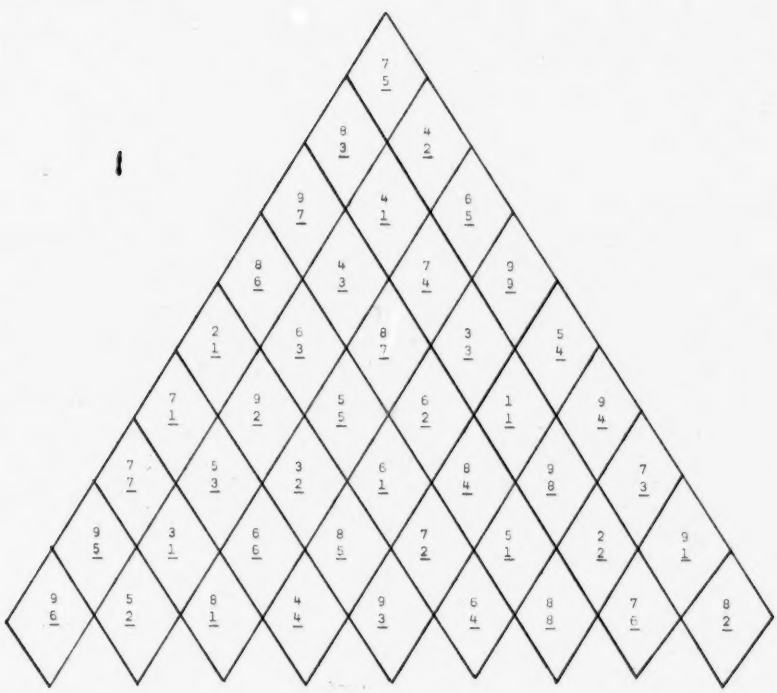

Figure 2. The 45 addition facts

Using Subtraction

When adding numbers that end in 7, 8 or 9, there may be advantages in increasing the units digit to 10, then subtract from the sum the difference between the original units digit and 10. When adding 29 to 45, think 30 + 45 = 75, then subtract 1 (30 - 29) to arrive at the answer of 74. When combining numbers such as 65 and 98, think 65 + 100 = 165 less 2 (100 - 98) equals 163. This technique can be practiced using problems similar to the following:

75	78	197
97	29	46

Proving

The most common method of proving addition is to add the columns in reverse order. If the figures have been copied from another source, they should be compared with the original numbers. Pupils should be aware that this is also an important task when using an adding machine. Another method for proving addition, variously referred to as the "accountant's method" or the "civil service method," is to total each column separately without carrying a figure into the next column as illustrated subsequently. This is an excellent method to use for checking the accuracy of a long column of figures or for initial addition when one may be interrupted frequently during his calculating.

245	
38	
122	
476	
73	
359	
228	
47	
185	
633	
154	
288	
365	
56	
341	
80	sum of unit's column
73	sum of ten's column
28	sum of hundred's column
3,610	

Another way of dealing with this problem is to break a column of numbers into subtotals by drawing a line under every five numbers as illustrated on the left or by dividing the long columns into several shorter problems and then calculate the grand total from the individual totals also as illustrated on the right.

245		
38		
122		
476		
73		
359	954	subtotal
228		
47		
185		
633		
154	2,406	subtotal
288		
365		
56		
341		
	3,610	total

245		
38		
122		
476		
73	954	individual total
359		
228		
47		
185		
633	1,452	individual total
154		
288		
365		
56		
341	1,204	individual total
	3,610	total

Some problems in business have built in checks such as is found in Figure 3. This built in check may fail if a poorly written number is incorrectely read when calculating both vertically and horizontally.

SALES BY DEPARTMENTS

Department	Monday	Tuesday	Wednesday	Thursday	Friday	Totals
A	248.10	326.35	309.19	422.67	562.00	1,868.31
B	115.89	189.98	199.15	236.50	278.95	1,020.47
C	265.07	245.33	316.12	401.19	398.43	1,626.14
TOTALS	629.06	761.66	824.46	1,060.36	1,239.38	4,514.92

Figure 3. Sales summary

Multiplication

Multiplication is the second most frequently performed calculation in personal and business situations. It is a short way of adding a given num-

ber numerous times; the multiplier indicates how many times the multiplicand is to be added.

Developing Understanding

To understand the algorithm of multiplication, pupils should be aware of the place-value notation of the decimal system. A consciousness of place value can be developed by periodically questioning pupils as to the value of the digits in a number. In the problem 256 × 134 the teacher could direct the following questions concerning the multiplier: "Does the *1* stand for *one, ten* or *one hundred?*" "Does the *3* stand for *three, thirty* or *three hundred?*" Does the *4* stand for *four, forty* or *four hundred?*" The problem then should be placed on the chalkboard or a transparency where the teacher can illustrate how the partial products are placed in relationship to the place values of the digits in the multiplier.

256	
× 134	(100 + 30 + 4)
1024	(256 repeated 4 times, partial product placed in unit's column)
768	(256 repeated 3 times, partial product placed in ten's column)
256	(256 added 1 time, partial product placed in hundred's column)
34,304	

Colored chalk may be used when developing the concept of place-value notation. In recording the answers to the respective multiplications in the previous problem by 4, 30 and 100, the moving over of one place to the left, which is done by placing the 8 under the 2, can be illustrated by placing a red zero (0) under the 4 indicating the teacher has multiplied by 30 and not by 3. In the following illustration the italicized zeroes would be written in red (or some other available color) on the chalkboard or on a transparency.

256	
× 134	
1024	
768*0*	(one red zero to indicate ten's)
2560*0*	(two red zeros to indicate hundred's)
34,304	

Another method currently being used to develop understanding of the place value of numerals is illustrated subsequently. Here, not more than one period should be spent on breaking down multiplication problems

into component parts according to place value. This may help pupils understand the values of digits in relationship to their placement in a number. The sample problem 256 × 134 would be structured as illustrated in Figure 4.

```
  200 + 50 + 6      multiplicand (256)
x 100 + 30 + 4      multiplier   (134)
                24 ┐
               200 ⟩ 1024    partial product (256 x 4)
               800 ┘
               180 ┐
              1500 ⟩ 7680    partial product (256 x 30)
              6000 ┘
               600 ┐
              5000 ⟩ 25600   partial product (256 x 100)
             20000 ┘
             34304    34304  product
```

Figure 4. Understanding of place value

The process of repeated addition can be illustrated through the use of an adding machine tape. Using the sample problem 246 × 35, the number "246" could be repeated on the adding machine 35 times, with the total of 8,610 resulting. Pupils might then be asked if they could devise a shorter method for use with the adding machine. The sample problem could be placed on the chalkboard or a transparency for overhead projection. A comparison should then be made between the adding machine method and the manual method used by most persons doing pencil and paper calculations as shown in Figure 5.

```
Adding Machine Method                                  Manual Method

 246 ┐
 246 │
 246 ⟩ 5(246)          = 1230   unit's total            246
 246 │                                                x 35
 246 ┘                                                1230
2460 ┐                                                738
2460 ⟩ 3(10 x 246)     = 7380   ten's total            8610
2460 ┘
8610                     8610
```

Figure 5. Comparison

These illustrations should help pupils develop insight into multiplication as a short cut to repeated addition, with place value being of utmost importance. One illustration may be sufficient for some classes while others may require additional presentations of this type before most pupils gain insight into the multiplication process.

Placing Decimals

Usually the placing of decimals in multiplication problems does not present difficulty to pupils. Some business mathematics textbooks provide illustrations which explain why the total number of decimal places in both the multiplier and the multiplicand is marked off in the product. These explanations are especially helpful to those pupils who have an inquiring mind. Pupils need to be encouraged to estimate their answers in terms of the whole numbers involved to determine if the decimal has been properly placed.

Estimating

To obtain a close estimate in multiplication the teacher should advise pupils that, whenever possible, increase one factor and decrease the other.

578	Estimate:	600	multiplicand increased
× 321		× 300	multiplier decreased
578		180,000	estimated product
1156			
1734			
185,538	actual product		

In problems where it is not advisable to increase one factor and decrease the other both factors are either increased or decreased, the estimated answers may not be as close to the actual answers as may be desired.

When estimating the answers in multiplication problems containing decimals, pupils should be taught to drop the decimal fraction as shown in the following example:

436.25	Estimate:	400	
× 28.75		× 30	
218125		12,000	estimated product
305375			
349000			
87250			
12,542.1875	actual product		

Pupils should be shown that whenever the multiplier is less than one, the resultant product will be smaller than the multiplicand. In the following the multiplier does not contain a whole number; therefore, the estimation may be made by taking a fractional part of the multiplicand.

```
  43.78                          Estimate: 40 × ¾ = 30 estimated
× .72  (estimate as ¾)                   product
------
  8756
30646
------
31.5216 actual product
```

Proving

There are several methods which usually are taught for proving multiplication. One method (reverse method) is to transpose the multiplicand and the multiplier. This is probably the best method when the multiplication factors contain the same or almost the same number of digits. Another method (division method), used especially when the multiplier is small, is to divide the product by the multiplier, and the quotient should equal the multiplicand. A third method (casting out 9's) also is advocated by some business mathematics textbooks. Figures 6, 7 and 8 are illustrations of these methods.

```
  328          216
× 216        × 328
------       ------
 1968         1728
 328          432
 656          648
------       ------
70,848       70,848
```

Figure 6. Reverse method

```
  4,265          4,265
×   17       17) 72,505
------         68
 29855         ---
 4265           45
------          34
72,505          ---
                110
                102
                ---
                 85
                 85
                ---
```

Figure 7. Division method

```
 247   cancel figures totaling 9; leaves a remainder of 4
x 68   6 + 8 = 14; 1 + 4 = 5
1976         4 x 5 = 20; 2 + 0 = 2
1482
16,796 1 + 6 + 7 + 6 = 20; 2 + 0 = 2
```

Figure 8. Casting out 9's method

In the casting out of 9's method, the remainder of 2 in the combined multiplicand and multiplier equals the remainder of 2 in the product. This indicates probable correctness of the answer.

Other Methods

In proving multiplication, for an enrichment exercise or for motivational purposes, teachers may wish to introduce one or more interesting methods of multiplication. The lattice method and the halving and doubling method are illustrated on the following pages, and a reference source is provided for making and using Napier's rods.

Lattice Method. This method of multiplication has been handed down through the ages. It is a simple method permitting a person to multiply only one set of digits at a time with no carrying required during the multiplying phase. After multiplying all sets of digits, the addition is performed diagonally. There are three steps in working multiplication problems in the lattice method, and they are illustrated subsequently using the sample problem 358×42.

Step 1. Diagram the Problem. The number of blocks to be drawn depends on the number of digits in the multiplicand and the multiplier. There should be a vertical column of blocks for each digit in the multiplier, as illustrated in Figure 9.

Step 2. Multiply the Individual Factors. Beginning in the upper right block, multiply the factors 4×8 and enter the product 32 in that block. The *ten's digit,* 3, is written above the diagonal and the *unit's digit,* 2, is placed below the diagonal. Working across the top row from right to left, multiply 4×5 and 4×3 and place the products in their respective blocks. Then take the next figure in the multiplier, 2, and multiply 2×8, 2×5 and 2×3 and place their products in the respective blocks, as illustrated in Figure 10.

Step 3. Perform Addition. The addition is performed by adding the numbers in the diagonal columns, represented by broken lines. If a ten's digit is to be *carried over,* it is placed at the top of the first block

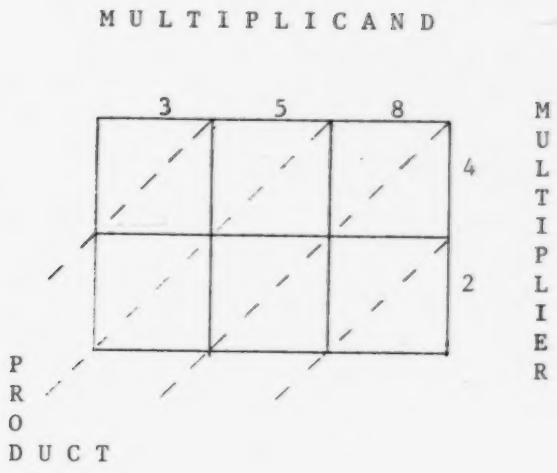

Figure 9. Diagram the problem

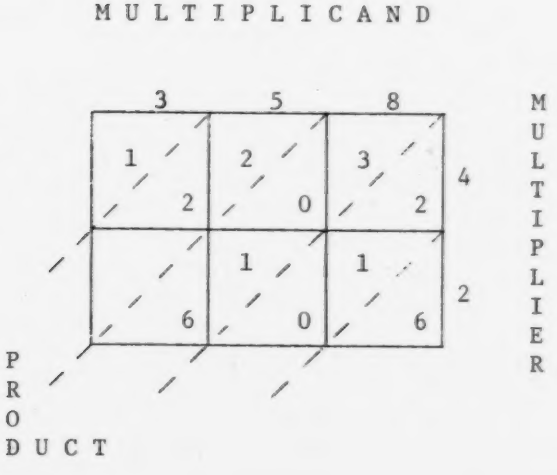

Figure 10. Multiply the individual factors

in the next diagonal column to the left and circled so it stands separate and apart from the other digits. See Figure 11.

Halving and Doubling Method. The halving and doubling method of multiplication involves halving the multiplicand and doubling the multiplier. This method may be explained in the following steps:

1. Identify the factors to be multiplied.
2. Halve the one factor progressively until arriving at the answer of 1. Drop any resulting fractions during the process.
3. Double the other factor as many times as you halved the first factor.
4. Locate all even numbers resulting from halving the first factor and cross out the even numbers and the corresponding doubled numbers.
5. The sum of the remaining doubled numbers is the product of the factors.

Using the halving and doubling method of multiplication, the solution to the sample problem 45×25 would appear as shown in Figure 12.

Napier's Rods. The previously described lattice method is similar to a set of numerating rods, sometimes referred to as "Napier's bones," invented by John Napier, a Scottish mathematician, in 1617. A set of Napier's rods can be simulated on cardboard and used to add interest and variety in solving or checking solutions to multiplication problems.

A project of this type might serve as a motivational device for those less interested in learning to improve their multiplication skills as it provides a manipulating tool constructed by the learner. Its suggested use in this instance would be for checking solutions to assigned problems previously worked with pencil and paper. Conversely, this project could be used as an enrichment exercise for those pupils who have demonstrated a high level of ability in multiplication and are exempted from the general assignment given other pupils. The enrichment exercise might also include a short oral or written report on John Napier and a class demonstration on how the rods are used.

A booklet by Glenn and Johnson[1] provides complete, easy-to-understand instructions and illustrations for making and using a set of Napier's rods. This reference source also contains information and illustrations on the abacus, slide rule, nomograph and binary system. In addition, a brief history of counting devices and calculating machines is included.

[1] William H. Glenn and Donovan A. Johnson, *Computing Devices,* (St. Louis: Webster Publishing Company, 1961), 55 pp.

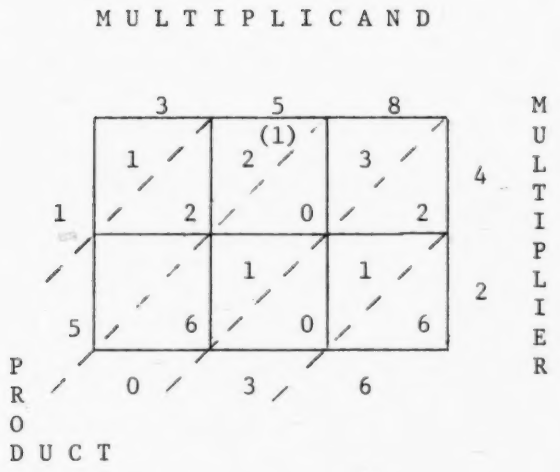

Figure 11. Perform addition

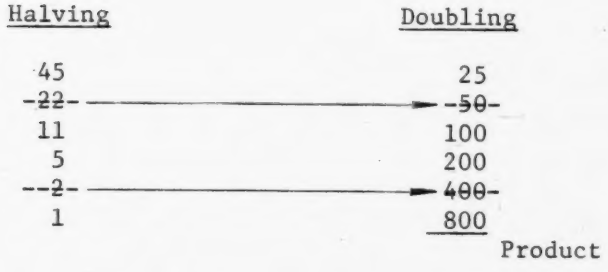

Figure 12. Halving and doubling method

Subtraction

Pupils probably have learned subtraction by either the additive or the take-away method. Considering the problem 8 – 3, pupils using the additive process would think 3 plus what number equals 8; whereas, pupils using the take-away method would think 8 take away 3 leaves 5. A determination has not been made that one method is better than the other, nor is it advocated that pupils be expected to change the method they have been using since their elementary years. The method each pupil learned in prior years should be developed through exercises provided by the teacher. To further develop pupils' insight into the subtraction process, teachers may wish to consider presenting the borrowing concept and the complement concept.

Borrowing Concept

The regrouping of numbers can be used to help pupils develop insight into the borrowing concept as illustrated in the following problem.

53	50 + 3	40 + 13	(53) minuend
- 28	- 20 + - 8	- 20 + - 8	(-28) subtrahend
		20 + 5	(25) difference

Since 8 units cannot be taken away from 3 units (or using the additive approach, 8 units cannot be added to a number to arrive at 3 units, with the exception of a negative number), it is necessary to borrow one 10 from the 50 to add to the units digit, making it possible to subtract 8 from 13.

Complement Concept

Some pupils may find the complement concept—using the complement of 10—helpful in developing skill in subtraction. This method is used when the minuend is smaller than the subtrahend. The following problems are given to illustrate this procedure.

13	15	17
- 7	- 8	- 9

In the first problem on the left a pupil should think 13 minus 10 equals 3; then the difference between 10 and 7 is 3 more; adding this 3 to the first 3 provides an answer of 6. These problems are solved in the following manner:

13 − 7 think: 13 - 10 = 3
10 - 7 = 3
6

15 − 8 think: 15 - 10 = 5
10 - 8 = 2
7

17 − 9 think: 17 - 10 = 7
10 - 9 = 1
8

Proving

Experience has indicated that the simplest and most accurate method of proving subtraction is to add the difference and the subtrahend to arrive at the minuend. This can be done quickly and without the necessity of rewriting the problem or of having to work with other figures.

Some mathematics textbooks use two terms to refer to answers received in subtraction problems. The term *difference* is applied when the answer refers to how many more items are needed to complete a group or set or when comparing two numbers, and the term *remainder* is used when the answer refers to what is left after a number has been taken away from a group or set. In business mathematics the term to be applied in a subtraction problem often depends on the numerical data to which it applies. Terms such as *balance, proceeds* and *net amount* apply to their respective problems.

Division

As commonly taught, division involves both multiplication and subtraction; however, it is solved through a series of repeated subtractions.

Developing Understanding

Using the sample problem 3,840 ÷ 25, the following statement explains the division algorithm for the purpose of developing insight into this process: *The number of times that the divisor of 25 can be subtracted from the dividend of 3,840 is called the quotient, and any number less than 25 that remains is called the remainder.* This statement can be illustrated on the chalkboard or on a transparency as follows:

	153	quotient = (100 + 50 + 3)
divisor	25) 3,840	dividend
	-2,500	= 100 × 25 or (25 subtracted 100 times)
	1,340	
	-1,250	= 50 × 25 or (25 subtracted 50 times)
	90	
	-75	= 3 × 25 or (25 subtracted 3 times)
remainder	15	= 153 quotient (the number of times 25 has been subtracted from the dividend of 3,840)

The importance of place value in the decimal number system can be stressed again by questioning pupils on the value of the numbers in the quotient 153. The necessity of keeping figures in correct columns can be illustrated as follows:

```
  153          (100 + 50 + 3)
25) 3,840
   25
  134
  125
   90
   75
   15
```

Placing Decimals

Determining and placing decimal points in division problems constitutes a stumbling block for some learners. When reviewing the division of numbers containing decimals, pupils should be taught to *place the decimal first*. If the decimal point is determined and placed on the quotient line before the pupil begins dividing, he is less likely to misplace the decimal in the quotient. Again, the importance of keeping figures aligned needs to be stressed. The division problem in Figure 13 illustrates the steps to be taken when decimals are present.

Problem	Procedure
2.5)345.60	2.5)345.6,00

Figure 13. Placing decimals

Shortcut Method

Practically all business mathematics textbooks include the shortcut method for dividing by tens, hundreds and thousands. Pupils should be allowed to "discover" this method, thereby developing insight into why this method works. This can be accomplished by giving pupils five problems each having a divisor of 10 to be worked by long or short division. After all the problems have been solved, pupils are asked to examine each answer to see if a particular pattern is discernable in the quotients obtained. As pupils detect the pattern, each learner should be given an opportunity, with textbooks closed, of determining a shortcut method of solving these problems in the future and of writing a sentence describing the procedure. The next step is to assign five problems using 100 as a divisor and to follow the same procedure.

Estimating

One of the ways to estimate an answer to a division problem is to round off the divisor and the dividend. Pupils should be advised that the estimate will be more accurate if both the divisor and the dividend are increased or decreased. If one is increased and the other decreased, the estimate will not be as close to the correct answer as would be desired. Another method of estimating division is the use of a table of reciprocals whereby the dividend is multiplied by the decimal equivalent of the divisor.

Proving

While division is referred to as the process of repeated subtraction, the proving of answers to division problems involves the reverse process—repeated addition (multiplication). If a remainder doesn't occur in the solution to a division problem, the process simply involves multiplying the divisor times the quotient. If a remainder is present, the following formula—which should be elicited from the class by allowing pupils to do some discovering—is used.

$$divisor \times quotient + remainder = dividend$$

Not more than one method of proving division should be presented to a class. Some pupils may have learned the method whereby the dividend is divided by the quotient to arrive at the original divisor providing no remainder is involved. Another method is the casting out of 9's.

Developing Accuracy and Speed

Developing accuracy and speed in mathematics are interdependent. While accuracy should receive the greater emphasis, speed also needs to

be stressed. As pupils develop accuracy in arriving at correct answers, confidence develops and speed increases. As speed is developed, pupils usually compute more accurately because of having learned to use combinations.

Types of Drill Presentations

While repetitive practice drills are important in developing computational skills, they need to be skillfully presented to keep pupil's interest from diminishing. Exercises should be short, given frequently, aimed at a specific goal and presented in a variety of ways.

Exercises can be presented by the teacher, by recordings, by flash cards, by transparency and tachistoscope projections or by duplicated and printed worksheets. The same exercises might alternately be used for both accuracy and speed development. The methods to be used in working the exercises and the goals to be pursued should be explained adequately so all pupils are aware of them. For instance, one practice session may be devoted to developing speed in addition by using combinations totaling 10, while at another time drill may be focused on developing accuracy in subtraction when 9's appear in the minuends of drill exercises.

Oral Presentations

Oral drills can provide practice for pupils in developing listening skills, in performing mental calculations and in writing numbers neatly and legibly. Although teachers may directly dictate oral drills, some of the drills should be prerecorded on tapes. These tapes may be retained and used for several groups if the teacher has more than one class of business mathematics. Also, the teacher may wish to repeat the same drills periodically to check on skill development. In addition, disc recordings containing drills in the fundamental processes may be purchased. All types of mental and written problems may be presented orally to a class.

Recitations. Teachers need to encourage all pupils to participate actively in exercises involving mental calculations. This is especially critical when individuals are called upon to recite. To attain as close to 100 per cent participation as possible, the following procedures are suggested:

1. *Dictate a problem or designate a printed problem.* If the problems are designated from a printed source on the pupils' desks, select the problems in random order; otherwise, as soon as a particular problem has been assigned to an individual, many of the other pupils might work ahead on the next problem rather than give their attention to the exercise at hand.

2. *Allow sufficient time for every class member to calculate the answer mentally.*

3. *Call upon pupils to recite the answers.* When pupils are called upon in a discernible pattern, many may not work the exercises seriously until their turn is apparent. After several pupils have recited, occasionally call upon individuals who have previously recited. Pupils will soon realize that they are subject to be called upon at any time.

Written Applications. Written problems may be presented orally to provide practice in writing numbers from oral dictation as well as to provide practice in the fundamental processes. Oral dictation of numerical data may occur on the job by way of telephone, intercom or personal conversation. An exercise to acquaint pupils with the importance of writing numbers accurately and legibly is to dictate problems to the class. When the dictation has been completed, the classmates use their neighbors' written data to perform the necessary calculations. The teacher may wish to explain that much of the written numerical data in business offices must be read and interpreted by other employes. To add realism to an exercise such as this, duplicated business forms might be used.

Increasing and Reducing Drills. Increasing and reducing drills can be used to add variety in practicing fundamental operations orally. Pupils start from a given number and increase or reduce that number by an indicated amount. For example, the teacher might say, "starting with the number 13, add 9's" or "beginning with the number 26, increase it by 7's" or "starting with the number 17, count off by 8's." For practice in subtracting, the teacher might say, "beginning with the number 256, subtract from it by 12's" or "starting with the number 342, reduce it by 9's" or "begining with the number 271 subtract 7's."

Visual Presentations

When projected materials are used, the drills should be previewed by having the class work through the projected problems in unison, after which individuals might be called upon to recite and to be timed in their calculations. Also, duplicated sheets containing the same problems could be distributed with each pupil recording the time it takes him to complete the problems—working, of course, for improved accuracy and speed.

Flash Cards. Flash cards can be used for group participation or for individual recitation. These cards may include addition, subtraction, multiplication or division exercises and pertain to drills with fractions, decimals or percentages. Pupils can make their own sets of flash cards

from the teacher's master set to be used in independent study. Index cards cut in half to 3 by 2½ inches are suggested for individual use. In addition to individual problems on flash cards, a series of numbers can be flashed with instructions for pupils to add 9's to them or to subtract 7's from them, etc. This same set of cards could be used for adding and subtracting other numbers, including two-digit numbers as pupils' skill develops.

Transparencies. The chief advantage of exercises on the overhead projector is that the attention of each pupil is focused on a single problem by using an ordinary sheet of paper to mask out the other exercises on the transparency. Transparencies are easy to prepare, easy to store and available at a moments notice for projection on screen or wall.

Tachistoscope Projections. Commercial filmstrips are available for projection by special tachistoscope projectors. Problems are projected on a screen at specific timed intervals which are preset. As pupils develop facility, the timed intervals are shortened. Some studies have indicated that tachistoscopic projections are a superior method for developing skill; other studies indicate the other methods previously mentioned can be as effective.

Duplicated and Printed Materials

Several excellent workbooks which can be used for building competency in the fundamentals of mathematics are available from business education textbook publishers. In addition, teachers may desire to develop and duplicate some of their own ideas. To obtain optimum use from a fundamentals workbook, teachers may instruct pupils to refrain from writing the answers to certain exercises in their workbooks. These exercises may then be used repeatedly throughout the course to develop and maintain accuracy and speed in computational skills. If a large paper cutter is available in the school, the business mathematics teacher may ask to borrow it to cut one or more reams of inexpensive white paper into one-, two- and three-inch strips to be used in conjunction with the workbooks. The one-inch strips can be used for writing the answers to addition, subtraction and simple multiplication and division problems by placing the strip directly below a horizontal row or directly along side a vertical row of exercises. The two- and three-inch strips may be used for making pencil calculations of more involved multiplication and division problems.

Some of the duplicated exercises might include answers. Pupils may use strips of cardboard to cover the printed answers. After the mental calculation is made for a particular problem, the strip may be removed to reveal the answer. This gives pupils immediate responses as to the

correctness of their mental calculations and is similar to the methodology used in teaching machines. This type of reinforcement is motivating and gives the added impetus to some pupils to complete the skill-building exercises.

Pupil Progress

Pupils need to be stimulated in some manner to strive for greater accuracy and speed in the fundamental skills. One way is to provide instances whereby competence in computation will help them throughout life in activities such as managing personal finances and working in the business world. However, these are distant, long-range goals and do not stimulate all young people. Pupils need present, short-range objectives, which may be provided by individually establishing goals for surpassing previous performances with basic computations.

One way this might be done is to have pupils maintain records or charts of their accuracy and speed performances for periodic drills on the same and/or similar drills. These charts will provide pupils with a record of their progress. Successful football coaches use this method—by timing players during various maneuvers—to get them to perform at their best. Unless a record is made of each individual's achievement, there is no basis upon which improvement can be checked. If a pupil can be shown by his individual record that he needs to work harder to improve or that his hard work is resulting in increased skill, he is more likely to respond in a positive way.

Checking Correctness of Figures

Pupils should be encouraged to develop the habit of checking copied figures against their source. They need to be aware that transpositions, omissions and illegible handwriting are common sources of errors. Exercises, similar to the ones found in state and federal civil service examinations may help pupils develop skill in recognizing transpositions and provide them with a familiarization of the types of numerical data they will need to understand during civil service examinations.

The learning of this aspect of business mathematics cannot be left to chance; the procedure of checking figures needs to be emphasized, taught, practiced and tested. Instruction needs to be planned, practice materials developed and test items constructed to insure that competence in this objective is attained.

A type of exercise for checking figures is to have pupils copy problems from both written and oral sources. Papers are then exchanged with classmates for checking purposes using the identical sources—oral and written. The main objective is to get pupils in the habit of checking numerical data which they are using for computational purposes against the original source to be sure the figures are correct.

Mathematics Shortcuts

While some shortcuts are useful, others tend to confuse pupils and cannot be justified for inclusion in this subject. Experience indicates that pupils with superior ability can grasp and use shortcuts to advantage while pupils of marginal ability frequently encounter difficulty with them. Pupils should master a few shortcuts rather than be familiar with many.

Conclusion

To develop competence in performing the fundamental processes, pupils need to: (1) recognize the value of computational skills; (2) be aware of the necessity for practice; (3) understand numbers and their relationships; (4) comprehend the purpose for which the exercises were developed; (5) be knowledgeable of the methods of practice to be employed; (6) perform practice drills thoughtfully and with a desire to improve; (7) be reinforced with correct responses and (8) be informed of their progress.

CHAPTER 4

FRACTIONS, DECIMALS AND PERCENTS

The use of fractions is unavoidable in business transactions, from computing the rate of interest on a bank deposit to the purchasing of stocks that are quoted in fractional parts of a dollar . . . there are few, if any, business transactions in which the decimal—or decimal fraction, as it is frequently called—does not appear . . . unless you understand percent, you may be fooled by the overenthusiastic advertisement . . .

—R. Robert Rosenberg and Harry Lewis

Insight into the meaning of fractions, decimals and percents as parts of a given number, and in some cases as ratios, needs to be developed. Also the relationships among fractions, decimals and percents are important considerations. Graphics may be used to illustrate the commonality of these mathematical concepts. Teaching procedures for developing the ability to understand and work with fractions, decimals and percents are presented in this chapter. The techniques for developing computational skill presented in the latter part of Chapter 3 are also applicable to the subject matter in this chapter.

Fractions

Pupils need to develop a clear concept of fractions and what they represent. When developing this concept, the numerators and denominators should be given attention separately. This can be accomplished by considering fractions as ratios consisting of two separate parts related to each other. Confronted with limited time, teachers sometimes resort to providing pupils with tailor-made rules for working with fractions rather than developing insight into their meaning. Rules per se are often necessary; however, they have a tendency to provide pupils with a crutch for calculating by rote memorization. Without understanding and insight, pupils usually have difficulty in intelligently applying fractions to problem solving situations.

In the business office, fractions usually are converted to decimals so problems can be worked on adding and calculating machines. Pupils should learn the decimal equivalents for those fractions frequently

encountered in the business world such as 1/2, 1/4, 1/8, 1/5, 1/10 and and 1/3. When pupils have mastered these few fractions, they can be shown how to figure 3/8 when 1/8 is known ($3 \times .125 = .375$). Pupils also need practice in using decimal equivalent charts in problem solving.

Developing the Fraction Concept

A fraction indicates a ratio or the comparison of a numerator to a denominator. Using the fraction 5/8 as an example, visualization of the fraction can be developed in the manner illustrated in Figure 14.

$\frac{5}{8}$ numerator
denominator

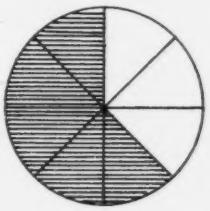

Figure 14. Fraction concept

Pupils need to understand thoroughly the italicized terms in the following statements:

1. A *fraction* represents a part of a whole.
2. The *numerator* is the *numberer,* which indicates the number of *parts.* It may be found by counting the number of shaded parts selected from the whole.
3. The *denominator* is the *namer,* which indicates the *name* or *total number of equal parts* which make up the whole. It is found by counting the total number of equal parts of which the whole consists.
4. The slash or horizontal bar of a fraction indicates *division.* For example, 5/8 indicates the division of 5 by 8. If the division were carried out, the decimal equivalent of .625 would result.

The teacher should then ask the class to make a verbal statement explaining the fraction 5/8. An answer similar to the following could be elicited from the class: *The 5 indicates the number of parts which have been selected from a whole consisting of 8 equal sections.*

Using the analysis approach to learning as described in the next chapter, the teacher should direct the class into understanding the following concerning fractions:

1. If the value of the whole is known, the value of one part or a group of parts can be calculated.
2. If the value of a group of parts is known, the value of 1 part can be calculated.
3. If the value of one part is known, the value of a group of parts or of the whole can be calculated.

If pupils can understand and visualize the preceding operation principles, the working of problems involving fractions should not be too difficult. The use of graphics similar to the illustrations in Figure 15 will help pupils grasp an understanding of how fractions are determined. In each instance in this illustration pupils should have developed the ability to observe that two parts have been selected from a whole consisting of three equal parts. In addition, pupils should be aware that comparison of numbers can be indicated as a division problem, $2 \div 3$; as a ratio, 2:3 or as a fraction, 2/3.

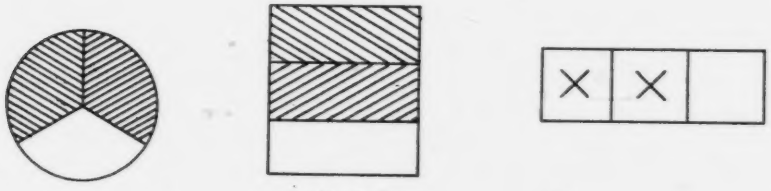

Figure 15. Developing insight into fractions

For those pupils who find working with fractions difficult, a basic exercise consisting of graphic illustrations of fractions may help them develop understanding of the fraction concept. Sentences asking pupils to write the fraction indicated also can be used for developing insight. Some examples are:

1. Mark had 3 hits out of 4 times at bat.
2. The football team won 7 out of 10 games this season.
3. Robert puts half of his allowance in a savings account.
4. It rained 12 days during the month of April.
5. Only 4 of the 12 apples that Barbara purchased were ripe enough to eat.
6. Douglas fired the rifle 32 times and hit a bull's eye 12 times.
7. Brenda attends school 5 days a week.

8. Joyce is willing to pay three-fifths of the cost of the party.
9. A fourth of the class was absent.
10. Of a total of 25 pupils in the business mathematics class, 15 were boys.
11. In a litter of 8 pups, 3 had white spots.
12. One out of every 6 peaches had blemishes.

Arriving at Common Denominators

In the sample addition problem of 1/4 + 1/3 + 1/6, an analogy can be made with the example that one can't add 4 apples, 3 peaches and 6 pears and arrive at 13 apples. The same thing is true in adding numbers and fractions in mathematics. Numbers must be added to like numbers. Graphic presentations, as illustrated in Figure 16, seem to be the best way to help pupils understand the concept of common denominators.

Reducing Fractions to Lowest Terms

The process of reducing fractions to their lowest terms may have been learned as a mechanical process; however, through the use of graphics, a teacher can explain and illustrate the logic of this procedure. The illustration in Figure 17 can be used to demonstrate that 4/12, 2/6 and 1/3 represent the same quantity. Pupils might be asked why fractions are reduced to their lowest terms with the teacher seeking answers such as the following: "A person can calculate easier with smaller figures." "A person can better visualize the portion which the fraction represents when it is expressed in its lowest terms." "A person can comprehend 1/3 of an amount or quantity easier than he can understand 4/12."

Reducing fractions to their lowest terms is sometimes a laborious task as the greatest common factor (divisor of the numerator and the denominator) is not readily apparent, causing several reductions to be made. There is a process which can be made available for pupils use to calculate the greatest common factor, and it is presented as follows:

Problem: Reduce the fraction 24/256 to its lowest terms.

Procedure:

Dividend		*Divisor*		*Quotient*	*Remainder*
256	÷	24	=	10	16
24	÷	16	=	1	8
16	÷	8	=	2	0

24/256 ÷ 8/8 = 3/32

Problem: 1/4 + 1/3 + 1/6 = 9/12 or 3/4

$\frac{1}{4}$ $\frac{3}{12}$

$\frac{1}{3}$ $\frac{4}{12}$

$\frac{1}{6}$ $\frac{2}{12}$

$\frac{9}{12}$ or $\frac{3}{4}$

Figure 16. Arriving at common denominator

0 0 0 0 | 0 0 0 0 0 0 0 0

$\frac{4}{12}$

$\frac{4}{12}$ of 12 = 4

00 00 | 00 00 00 00

$\frac{2}{6}$

$\frac{2}{6}$ of 12 = 4

0000 | 0000 0000

$\frac{1}{3}$

$\frac{1}{3}$ of 12 = 4

Figure 17. Understanding fractions

The following is an explanation of the procedure: First, divide the larger number by the smaller. Second, the first divisor then becomes the dividend and the remainder serves as the new divisor. Third, repeat the dividing and transferring process until the remainder is zero. Fourth, the last divisor is the greatest common factor of the given number.

Aliquot Parts

Aliquot parts frequently are used in the business world as convenient short cuts in calculating certain multiplication problems. The term *aliquot* refers to any number that is contained in another number an exact number of times. The first number is known as an *aliquot part* of the second number, and the second number is referred to as the *base.*

While the term aliquot part could relate to almost any quantity, its most practical use is with a base of 100 cents (equaling $1) and with a base of 100 percent (equaling 1). Some textbooks also include units on aliquot parts of 10 and 100. In addition, the 60-day, 6 percent method of computing interest involves the use of aliquot parts.

Once pupils master the use of aliquot parts, they can save time in solving problems. The following method is suggested to motivate pupils in the study and use of aliquot parts.

1. The teacher walks to the chalkboard and writes the following problem: What is the cost of 540 yards of material at 16 2/3¢ per yard?

2. He then says to the class, "The answer to this problem is $90, and I was able to calculate that answer in less than ten seconds; however, I used a *trick*. Would any of you like to know the trick?"

3. The teacher states, "Certain unit prices are exact fractional parts of 100 cents ($1). That is, each of the prices are contained in 100 cents without a remainder; and they are known as aliquot parts of one dollar. For example, 16 2/3¢ is contained in 100¢ exactly 6 times. Therefore, 16 2/3¢ is 1/6 of $1. In the sample problem on the chalkboard, you will arrive at the same answer whether you multiply 540 × 16 2/3¢ or whether you multiply 540 × $1/6."

At this point, the teacher may want to multiply 540 × 16 2/3¢ on the chalkboard to show the work involved in the long method. Next, the teacher should check the pupils' understanding of the process by projecting on a screen, or writing on the chalkboard, problems containing the same unit price of 16 2/3¢. Individual pupils are called upon to mentally calculate the problems and to recite the answers. The following problems are suggested:

30 @ 16 2/3¢	360 @ 16 2/3¢
48 @ 16 2/3¢	240 @ 16 2/3¢

The next step would be to project or write additional problems containing a different unit price, such as 12 1/2¢. The following are suggested:

32 @ 12 1/2¢	96 @ 12 1/2¢
48 @ 12 1/2¢	328 @ 12 1/2¢

A pupil might ask how he can find the aliquot part of a number if a chart is not available. One method is to divide 100 cents by 12 1/2 cents to determine if the unit price is an aliquot part of the base as shown in Figure 18.

$$\frac{100}{1} \div 12\frac{1}{2} = \frac{100}{1} \div \frac{25}{2} = \frac{\cancel{100}^{4}}{1} \times \frac{2}{\cancel{25}_{1}} = 8$$

Figure 18. Finding aliquot part

Pupils should be encouraged to learn the aliquot parts of the most common units, and they should be taught to refer to a chart if they will be working extensively with aliquot parts. Charts of this type are widely used in the business world, and pupils should be introduced and encouraged to use them in class.

From this point, pupils may be given a chart listing the common aliquot parts of $1 and problems involving their use; or they may be assigned a textbook unit on aliquot parts if the teacher feels they thoroughly understand this process.

When working with aliquot parts of bases other than 1 (100% = 1.00 and 100¢ = $1.00), the teacher can present the principles subsequently listed in italic type. The following is given to illustrate the three principles:

Problem: The student store purchased 25 pennants at a unit price of 48¢. Calculate the total cost of these pennants.

Procedure:

1. *Select and test a base.* A base of 100 is selected because 25 will divide evenly into it. The base of 100 is then tested to be sure it will reduce into a common fraction.

$$\frac{25}{100} = \frac{1}{4}$$

2. *Multiply the multiplicand by the base.*

$$48¢ \times 100 = \$48.00$$

3. *Multiply the product from step 2 by the aliquot part.*

$$\frac{1}{4} \times \$48 = \$12 \text{ Cost of the pennants}$$

Decimals

Pupils may not be impressed by the seriousness of an error in placing decimal points. Most teachers have heard pupils remark, "The answer's all right except for the decimal point!" Exclamations such as this one imply an unawareness on the pupils' part of how serious the error is. They need to develop a sensitiveness that misplacing the decimal point gives an incorrect answer.

The placement of decimal points in addition or subtraction usually causes little difficulty if care is exercised in aligning numbers and decimal points when copying problems; however, errors occur frequently in placing the decimal point in multiplication and in division problems. Errors can often be traced to carelessness, to haste and to the lack of estimating the reasonableness of answers.

Developing Understanding

To develop competence in working with decimals, pupils need to gain insight into their values as fractional parts of wholes. The following sequence is suggested:

1. Provide illustrations showing decimal fraction values.
2. Demonstrate place value and its importance.
3. Provide exercises for reading decimal fractions correctly.
4. Develop skill in writing decimal fractions from illustrations, verbal sentences and oral dictation.
5. Provide exercises for calculating with decimals in the four fundamental processes.
6. Teach the process of estimating decimal placement.
7. Instruct pupils how to round decimals to the nearest one-tenth, one-hundredth and one-thousandth.
8. Assign exercises in changing fraction and percentages to decimals and vice versa.
9. Assign verbal problems involving decimal fractions.

The following procedure illustrates why there are as many digits in the fractional part of the product of two decimal fractions as there are in the sum of the digits in the multiplier and the multiplicand.

Problem: $28.35 \times 12.25 = 347.2875$

Procedure:

$$\frac{2835}{100} \times \frac{1225}{100} = \frac{3472875}{10000} = 347\frac{2875}{10000} \text{ or } 347.2875$$

A similar procedure, perhaps more difficult for some pupils to perceive, can be used to illustrate why the number of decimal places in the divisor affects and requires moving the decimal point in the dividend. The cancellation process between the 100's in the divisor and the 10,000's in the dividend illustrates this point.

Problem: $347.2875 \div 28.35 = 12.25$

Procedure:

$$\frac{3472875}{10000} \div \frac{2835}{100} = \frac{3472875}{10000} \times \frac{100}{2835} = \frac{3472875}{100} \times \frac{1}{2835} = \frac{3472875}{238500} = 12.25$$

The procedure of placing the decimal first in division problems may be found in Chapter 3.

Estimating Decimal Placement

When working with decimals, a quick method of checking the placement of the decimal point in a product or a quotient is needed. The sample problem 28.35×12.25, which yields the digits 3472875, requires the correct placement of a decimal point. Placement may be found by using the rule of combining the number of decimal places in the multiplicand and the multiplier. Estimating the answer also helps in checking the placement of the decimal point. Since 28.35 is a little less than 30 and 12.25 is slightly more than 10, the estimated product (whole number portion of answer) should be in the neighborhood of 300 (30×10). By checking the whole numbers of the product against this estimate, the accuracy of the placement of the decimal is apparent.

Placement of decimal points in division problems may be determined and checked by a similar process. For example, when dividing 347.2875 by 28.35, the sequence of digits appearing in the quotient are 1225. Since the divisor is close to 30 and the dividend is over 300, it is apparent that $300 \div 30 = 10$. The quotient, therefore, should be in the neighborhood of 10 plus, which reasonably corresponds to the answer of 12.25. Another method of checking the accuracy of the placement of decimal points in quotients is to multiply the whole number of the quotient, in this case 12, times the divisor rounded off to the nearest

tens, in this case 30. The estimated whole number of 360 (12 × 30) corresponds reasonably in place value with the dividend of 347.2875.

Percents

Percentage is used to interpret, compare and communicate many kinds of quantitative information, and it has widespread utility and direct application in the business world. The Latin words *per centum* are literally translated "by the hundred," and thus indicate that a per cent is a ratio of a number to 100.

As percentage is closely allied to the subject of fractions and decimals, its application should involve no difficulties that are not involved in the study or application of decimals; however, experience indicates this is not usually the case. Percentage presents a troublesome aspect of business mathematics.

Developing Understanding

If a clear understanding of the percentage symbol and its relationship to the numbers it accompanies can be developed, pupils' difficulty with calculations involving percentage may be decreased. The following aspects concerning percentage may be presented to and discussed with the class:

1. A number identified as a per cent represents a part of the whole.
2. In percentage, a whole is represented by 100 equal parts; therefore, twenty-five per cent or 25% indicates 25 parts of a whole containing 100 equal parts.
3. The per cent symbol, %, represents the decimal fraction .01; therefore, 25% is equal to the decimal fraction .25 (25 × .01).
4. If the value of one part, 1%, can be ascertained, the value of the whole or any fractional part of the whole can be calculated.

The concepts of base, rate and percentages and their relationships can be clarified by using graphic materials. Many textbooks employ this technique to a limited extent, but they need to be supplemented by materials from a resource file. These materials may include prepared transparencies, duplicated study guides and illustrations portraying percentages from newspapers, magazines and financial reports.

Pupils often have difficulty in recognizing the differences between 25% and .25% and between 125% and 1.25%. They need to be taught to examine and analyze the differences between these rates. If pupils can recall that the percentage symbol represnts .01 as previously stated in Item 3, they can easily calculate the decimal fractions and arrive at an accurate figure. For example:

$25\% = (25 \times .01) = .25$ $\quad$ $125\% = (125 \times .01) = 1.25$

$.25\% = (.25 \times .01) = .0025$ $\quad$ $1.25\% = (1.25 \times .01) = .0125$

Some pupils undoubtedly will recall from previous learning that it's not necessary to multiply a per cent by .01 to find the decimal fraction; they realize the necessity to move the decimal point two places to the left when changing percents to decimals. Other pupils may have learned to change a per cent to its fractional equivalent and then convert the fraction to a decimal. When understood, these methods are acceptable and should be permitted by the teacher providing pupils can arrive at correct decimal conversions with consistency.

In the previous listing, Item 4 is helpful when pupils know what value a certain per cent represents and need to find the value of 100 per cent or a decimal fraction different from the one known. For example: David Drum sold his 75% interest in a drive-in restaurant for $12,000. What is the total value of the restaurant? Through analysis, a pupil can determine what 1 of the 75 parts is worth. After the value of 1 part is known, the value of the whole (100%) is easy to determine. With this type of analysis, it is not necessary to use decimal points. Some textbooks refer to this as the *1 per cent method;* that is, *reducing any known per cent factor to 1 part from which it is easy to ascertain the whole or any part of the whole.* Using the example cited in this paragraph, the following calculations illustrate the 1 per cent method:

```
      160
   ----------
75) 12,000
      75
      ---
      450
      450
      ---
        00
        00
        --
```

$ 160 value of 1 part (1%)
× 100 parts equaling a whole (100%)
$ 16,000 value of the whole

A problem of this type can be proved by using a graphic representation as indicated in Figure 19 on the following page.

One Formula Approach

Business mathematics textbooks generally advocate that pupils learn the three basic formulas of *percentage = base × rate, base = percentage ÷ rate* and *rate = percentage ÷ base.* Butler and Wren[1] suggest using one basic formula, *percentage = base × rate,* to eliminate the difficulty that usually results from the use of three formulas—one

[1] Charles H. Butler, F. Lynwood Wren and J. Houston Banks, *The Teaching of Secondary Mathematics,* (New York: McGraw-Hill Book Company, Inc., 1970), pp. 212-218.

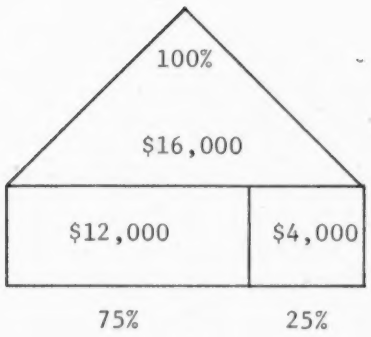

Figure 19. Graphic portrayal

for finding the rate, another for finding the base and still another for calculating the percentage.

When problems include the base and rate, the basic formula applies without much thought on the part of the pupil; however, when the percentage is given along with either the rate or the base, pupils are asked to make a decision on the basis of their knowledge of computational relationshpis. That is, when one factor is multiplied by another factor it results in a product. In turn, if that product is divided by either the multiplicand or the multiplier, the quotient will represent the other.

> . . . there are always three numbers involved [in percentage problems] . . . one of them [percentage] is the product of the other two, *base* × *rate*. If the product number percentage is not known, then the operation called for is multiplication; if it is known, then the operation called for is division.[1]

The following example uses the basic formula (a multiplication process) when the percentage is not known:

Percentage = *Base* × *Rate*

P = 125 × .12 (Percentage unknown)

15.00 = 125 × .12

[1] *Ibid.*, p. 215.

To prove a multiplication problem (or in this case when one of the multiplication factors is unknown) the division process can be used.

Percentage = *Base* × *Rate*

15.00 = 125 × R (Rate unknown—an unknown factor in a multiplication problem calls for division.)

15.00 ÷ 125 = .12 (Rate)

Percentage = *Base* × *Rate*

15.00 = B × .12 (Base unknown—an unknown factor in a multiplication problem calls for division.)

15.00 ÷ .12 = 125 (Base)

New Mathematics Approach

In certain new approaches in the teaching of mathematics, problems involving per cent are solved by eliminating decimal points and using the same procedure for all problems. Problems including per cent are set up in the form of an equation and an unknown factor can be computed by applying the following principles:

1. Equal fractions principle: *If two fractions are equal, the product of their opposites is equal.*

For example, consider the percentage problem 25% × $300 = $75 in fractional terms. If two fractions are equal, the products of their opposites are equal

$$\frac{25\%}{100\%} = \frac{\$75}{\$300}$$

$$25 \times 300 = 75 \times 100$$

$$7{,}500 = 7{,}500$$

In the preceding example, the ratio of 25% (rate—fractional part) as compared to 100% (whole) is equal to $75 (percentage—fractional part) as compared to $300 (base—whole). By expressing these ratios as fractions, the equal fractions principle can be applied.

2. Product of two factors principle: *If a number is represented by the product of two of its factors, then either factor divided into the product will give the other factors as a quotient.*

Using the percentage problem 25% × $300 = $75 and assuming that one of the factors is unknown in each of the examples, the following calculations illustrate these two principles:

Rate Unknown:

$$\frac{R}{100} = \frac{75}{300}$$

Principle 1: $R \times 300 = 75 \times 100$

$R \times 300 = 7{,}500$

Principle 2: $R = 7{,}500 \div 300$

$R = 25$

Base Unknown:

$$\frac{25}{300} = \frac{75}{B}$$

Principle 1: $25 \times B = 75 \times 100$

$25 \times B = 7{,}500$

Principle 2: $B = 7{,}500 \div 25$

$B = 300$

Percentage Unknown:

$$\frac{25}{100} = \frac{P}{300}$$

Principle 1: $25 \times 300 = P \times 100$

$7{,}500 = P \times 100$

Principle 2: $7{,}500 \div 100 = P$

$75 = P$

One problem which may arise when working with the new mathematics approach is when a rate such as 2.5% is given. When a rate containing a fractional part of one per cent is given, the pupils can be taught to multiply the given per cent rate by .01 representing the per cent symbol to establish the correct ratio and then convert the decimal to a fraction. Using the rate of 2.5% as an example, the following steps are suggested:

Step A: $2.5 \times .01 = .025$

Step B: $.025 = \frac{25}{1{,}000}$

The percentage problem 2.5% × $240 = $6 is used in the following presentation.

Rate Unknown:

$$\frac{R}{100} = \frac{6}{240}$$

Principle 1: $R \times 240 = 6 \times 100$

$R \times 240 = 600$

Principle 2: $R = 600 \div 240$

$R = 2.5$

When the *rate* is *unknown*, the ratio is always stated as R/100 as shown in the above example.

Base Unknown:

$$\frac{25}{1{,}000} = \frac{6}{B}$$

Principle 1:

$$25 \times B = 6 \times 1{,}000$$
$$25 \times B = 6{,}000$$

Principle 2:

$$B = 6{,}000 \div 25$$
$$B = 240$$

Percentage Unknown:

$$\frac{25}{1{,}000} = \frac{P}{240}$$

Principle 1:

$$25 \times 240 = P \times 1{,}000$$
$$6{,}000 = P \times 1{,}000$$

Principle 2:

$$6{,}000 \div 1{,}000 = P$$
$$6 = P$$

Another concern in the new mathematics approach occurs when either the percentage figure or the base figure or both include dollars and cents. Although not absolutely necessary, both the percentage and the base figures may be converted to cents as illustrated in the problem 25% × $85 = $21.25.

Equal Fractions

$$\frac{25\%}{100\%} = \frac{\$21.25}{\$85}$$

Restated As:

$$\frac{25\%}{100\%} = \frac{2125 \text{ cents}}{8500 \text{ cents}}$$

In the restated fraction the percentage of $21.25 is restated as 2,125 cents and the base of $85 is restated as 8,500 cents.

In analyzing the relationship of the new mathematics approach to applications in personal and business situations, this method's chief advantage would be for those problems in which the rate or the base are unknown. In many cases those problems are the type in which high school pupils experience the most difficulty. The following examples are illustrative of the types of problems in which either the rate or the base are unknown.

1. A salesman receives a commission of 7% on all sales. What amount of goods must he sell in one month to earn $875 in commisions?

Equal Fractions:

$$\frac{7}{100} = \frac{875}{B}$$

Solution:

$$7 \times B = 875 \times 100$$
$$7 \times B = 87{,}500$$
$$B = 87{,}000 \div 7$$
$$B = \$12{,}500$$

2. In Valley View 20% of the residents attend school. If 1,238 children and teen-agers are enrolled in school, what is the total population of Valley View?

Equal Fractions:

$$\frac{20}{100} = \frac{1238}{B}$$

Solution:

$$20 \times B = 1{,}238 \times 100$$
$$20 \times B = 123{,}800$$
$$B = 123{,}800 \div 20$$
$$B = 6{,}190$$

3. Steve had 85 hits out of 242 times at bat. What is his per cent of successful hits?

Equal Fractions:

$$\frac{R}{100} = \frac{85}{242}$$

Solution:

$$R \times 242 = 85 \times 100$$
$$R \times 242 = 8{,}500$$
$$R = 8{,}500 \div 242$$
$$R = 35.1\%$$

4. A publisher's representative solicited subscriptions for a monthly magazine which sold for $1.50 a year. If he receives 48 cents from the publisher for each annual subscription he sells, what rate of commission does he receive?

Equal Fractions:

$$\frac{R}{100} = \frac{.48}{1.50}$$

or

$$\frac{R}{100} = \frac{48 \text{ cents}}{150 \text{ cents}}$$

Solution:

$$R \times 150 = 48 \times 100$$
$$R \times 150 = 4{,}800$$
$$R = 4{,}800 \div 150$$
$$R = 32\%$$

Conclusion

The learning cycle for fractions, decimals and percents should follow the pattern of *understanding, practice* and *application.* While this chapter suggests methods for developing an understanding of fractions, decimals and percents, the latter portion of Chapter 3 presents techniques for providing practice for pupils to achieve competence in working with these mathematical tools.

CHAPTER 5

DEVELOPING PROBLEM SOLVING TECHNIQUES

The translation of given real-life situations into mathematical symbolism is considered the most useful tool in problem solving.
—Kenneth B. Henderson and Robert E. Pingry

Teaching pupils to develop problem solving techniques is a challenging task. Solving verbal problems requires intellect, as well as interest and desire, and involves understanding and insight to a much greater degree than computation.

A pupil's attitude toward mathematics is an important consideration. Does he look at a mathematical problem as a challenge or does he look at it with disdain? Developing a positive attitude toward mathematics and toward problem solving is probably the most important factor in developing this skill.

In business mathematics, exercises are often referred to as problems; however, in this chapter, the term *problem* refers to verbal statements which must be translated into mathematical statements before the necessary calculations can be made.

Three suggested problem solving techniques include the analysis approach, the graphic approach and the discovery approach. While these methods may be used singly, they are more apt to be used in combination, depending on the problem to be solved and the ingenuity of the teacher.

Analysis Approach

The first step in problem solving involves the reading and comprehending of the verbal material. For some pupils, this may require reading the statement several times. To comprehend the statement thoroughly, pupils need to be familiar with the vocabulary used, including business and mathematical terminology.

When a pupil understands the statement and knows what is being asked, he must recall mathematical principles, formulas or equations

which relate to the problem. He must then restate the verbal problem in mathematical terms, which is the crucial test in problem solving. Once the problem is stated mathematically, the pupil performs the necessary calculations in the proper order. In some problems there are advantages in visualizing the problem graphically as explained and illustrated in this chapter under the heading, Graphic Approach.

Finally, the communication of the answer is important. Pupils should be taught to state their answers correctly in terms of what is asked. For example, if a problem specifically requests a percent, the answer should not be given as a decimal; or if the problem asks for a dollar amount, the dollar symbol ($) should not be omitted from the answer.

To illustrate the analysis approach the following sequence using a sample problem is presented:

1. Read and reread the sample problem until it is understood.

   To avoid paying a penalty on his property taxes, Raymond Dixon wants to borrow $440 from Modern Trust Company by signing a note for 9 months at the rate of 7%. At maturity, what will be the total amount he must pay?

2. Recall the formula for finding interest.

   Interest = Principal × Rate × Time

3. Restate the problem in mathematical terms.

   Interest = $440 × .07 × 9/12
   Amount at Maturity = $440 + Interest

4. Perform the necessary calculations in the sequence suggested by the mathematical statement of the problem.

   $440 principal
   × .07 rate
   $30.80 interest (1 year)

   Interest for 9 months would be 3/4 times this amount or $23.10.

   $440.00 principal
   23.10 interest
   $463.10 maturity value

5. State the answer in correct terms.

   Answer: $463.10 maturity value

The labeling of figures as illustrated in item 4 will help pupils see the relationship between the mathematical statement of the problem and the calculations performed.

To develop skill in problem solving, teachers may want to spend time on teaching pupils to read verbal problems carefully. This can be accomplished in part by questioning pupils on terminology, on the

relevance of the numerical data presented and on the intent of the problem. The next step would be to have pupils restate the problem in their own words. After this objective has been met, teachers may assign a series of problems whereby pupils are requested to restate the problem in mathematical terms. Building skill on the individual steps in the analysis approach will help pupils develop confidence as well as competence in using this method of solving problems.

Graphic Approach

In the graphic approach pupils are taught to use some form of visual drawing to help them understand and solve mathematical problems. Graphic representations serve to help learners identify elements of problems and perceive relationships between numbers involved. The illustrations in the fourth chapter are indicative of the type of presentations which help pupils develop insight into mathematical concepts. Graphics also can be used in teaching problem solving. The following example is illustrative of this approach:

Sample problem: On October 1, the student store had merchandise in stock costing $300. During the month, $500 worth of merchandise was purchased; and on October 30, $200 worth of merchandise was still in stock. Find the cost of the goods which were sold.

Question: What factors are known?

Answer: Beginning inventory of $300
Purchases during the month of $500
Ending inventory of $200

Indicate these factors graphically: See Figure 20.

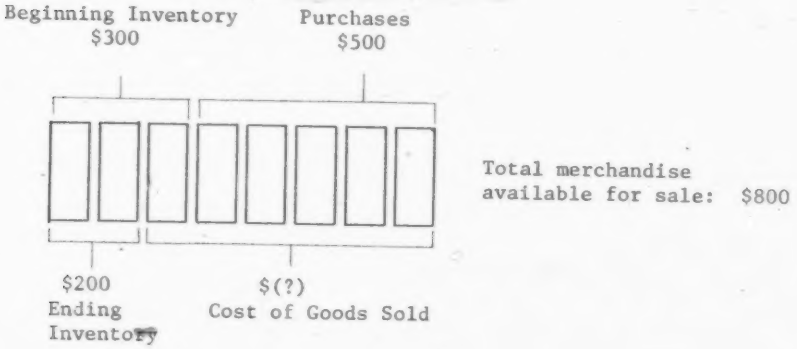

Figure 20. Graphic approach .

Question: Using this illustration, find the cost of the goods sold and develop a mathematical equation for use on similar problems.

Pupils can ascertain by this illustration that the cost of goods sold is $600; and they should write the following equation:

Beginning Inventory + *Purchases* = *Ending Inventory* + *Cost of Goods Sold*

Discovery Approach

In the discovery approach pupils are guided by pertinent questioning from the teacher and by recalling basic mathematical concepts that have been taught preceding the problem solving session. If pupils "discover" a problem solving process on their own, they are likely to remember it; and once forgotten, they will have more confidence in their ability to rediscover the solution. The discovery approach can best be presented by using a sample problem.

Sample problem: Richard Reinhardt purchased a 35% interest in a service station for $2,800. What is the total value of the service station?

Teacher: What is the formula for finding percentage?

Pupil: Percentage = Base × Rate

Teacher: How do the figures in the problem fit this formula?

Pupil: $2,800 = Base × .35

Teacher: What type of calculation is performed in this formula?

Pupil: Multiplication

Teacher: What are the factors in a multiplication problem?

Pupil: Product = Multiplicand × Multiplier

Teacher: How do the figures in the above problem fit these factors?

Pupil: $2,800 = Multiplicand × .35

Teacher: What are the two methods of proving a multiplication problem?

Pupil: By reversing the multiplier and the multiplicand or by dividing the product by the multiplier.

Teacher: Can one of these multiplication proof methods be useful in solving this problem?

Sufficient time should be allowed for pupils to observe the relationship between the percentage formula and the multiplication process and the relationship between the division method of proving a multiplication

problem and the process of finding the base in a percentage problem. Pupils' thought processes ought to reflect the following:

Multiplication Problem: \$2,800 = Multiplicand × .35

Proving and Multiplication Problem: Product ÷ Multiplier = Multiplicand

Discovery: Percentage ÷ Rate = Base

Application: \$2,800 ÷ .35 = \$8,000

Pupils can then be given additional written problems to solve using the "discovered" procedures. They also can be given the opportunity of applying this process to a similar problem stated in different terms. A problem such as the following might appear on an employment test and can be solved in the same manner: \$43.75 is 25% of what amount?

If the equal fractions principle, as explained in the previous chapter, is used to solve exercises of this type, pupils might be given problems to which this principle can be applied.

The discovery approach also can be used for homework assignments whereby the teacher provides the pupils with the facts and asks them to study the relationships. Although a time consuming process, this approach can be a valuable teaching technique when used with discretion.

Other Considerations

Other considerations in the teaching of problem solving include the use of reference materials, bulletin boards and models and patterns. The inclusion of charts and graphs, probability, statistics and binary system in a school's business mathematics course of study should be given some consideration. In addition, pupils should be aware of negative numbers and credit balances as discussed in Appendix C, *Understanding Negative Numbers.*

Reference Materials

Teachers can provide pupils with experiences in the use of reference materials in solving mathematical problems in business. Problems can be written which will require pupils to search in reference materials for data before they can solve them successfully. The following problems are illustrative of this type:

1. You are interested in buying a cuckoo clock from a Helvetian firm. The advertisement lists a price of 65 francs. Using a dictionary to find the English name of the country and the *New York Times* to find the foreign exchange rate (the teacher can post this page on the bulletin board or the pupil may be required to use the current edition in the school library), find out how much money in United States dollars you would have to send to purchase the clock.

2. Your boss is taking a trip to Germany to visit a subsidiary of the company for which you work. He is told that the plant is located 45 kilometers from the airport. You are asked to find the equivalent distance in miles from the airport to the plant. Using a dictionary to find the ratio between kilometers and miles, calculate the distance in miles.

Bulletin Boards

Bulletin boards can be used to illustrate a problem solving method, to present illustrations of mathematics usage in personal and business situations and to portray types of forms used in the business office requiring numerical data and/or computations. Whenever possible, pupils should participate in the preparation of the displays.

Models and Patterns

For those pupils who need to develop insight into some of the basic concepts of mathematics, models and patterns representing fractions, percents and decimals can be made from construction paper. The fractions, percents and decimals ought to be related to mathematical data within the high school.

Charts and Graphs

Charts are used extensively by office employees in many types of business transactions. While the use of charts can speed up the work flow, the misuse of them can be costly to a business in terms of goodwill and profit. In addition to the charts in a pupil's textbook the teacher may want to consider obtaining, projecting or duplicating additional types of charts for classroom use. Various types of charts can be obtained from business sources as well as from governmental agencies. The U.S. Internal Revenue Service provides income tax teaching kits which include withholding tax and social security tax charts. These charts are helpful when working with payroll problems. State sales tax charts also can be obtained.

Graphs can be used to add office atmosphere and increased interest to a business mathematics classroom. This unit could be started by having pupils search for examples of graphs to be posted on the bulletin board. After these examples are examined and discussed, the teacher may refer to the basics of reading and constructing graphs located in the textbook. The teacher then might ask pupils to suggest numerical data in the high school which might be presented graphically. Pupils could then be given their choice of data which they will be asked to present in one or several types of graphs. The best and most original graphs would be displayed in the classroom. Perhaps a photograph of the graphs and their makers may be the source of an article for the school's newspaper.

Probability

With the increased use of probability pupils should be introduced to this topic. Two booklets—*Chances Are* (a short introduction to probability) and *Sets, Probability and Statistics* (a more detailed study of probability)—are available free in classroom quantities. The sources of these publications may be found in Appendix B. In addition to using one or both of these publications, a teacher may consider having pupils conduct experiments with a coin or a die to compare predicted results with experimental results. Experiments of this type stimulate pupils and prepare them for a more serious study of the topic.

Statistics

Each pupil should understand the simple measures of central tendency —mean, medium and mode. One cannot read the newspaper today without finding these measures being used to express results of studies and opinion polls. Numerical data can be collected from sources within the school for the purpose of making a statistical analysis. These sources include school records such as tardiness and attendance data and registration figures. Student activities such as watching television, reading books, participating in sports and working after school are other sources of data which might be considered.

One project for the class to use in learning to calculate mean, median and mode is the recording of the school's basketball or football scores. An example using Camp Hill High School's basketball scores for the 1970-71 season is presented in Figure 21 on the following page.

After the mean, median and mode have been calculated for both the Camp Hill array and the opponent's array, the teacher can lead the class in a discussion about the meaning and significance of these three measures of central tendency. For the Camp Hill team, there are wide differences among the mean, median and mode. This could indicate that the offense was inconsistent—the scores ranged from a high of 106 to a low of 39. However, the defensive play of the Camp Hill team was more consistent than the offensive play, because the opponents' mean, median and mode are very close.

It would be difficult to guess how many points Camp Hill might score prior to a game. However, one could estimate the opponent's score to be about 46 points. In fact, 10 teams scored in the 40's and 15 teams scored between 36 and 56 or within 10 points of the average of 46.

Using a similar situation, a teacher can provide a learning experience from which a discussion of insurance or some other topic could continue.

Binary System

As indicated in the Department of Education's Bulletin 276, *Data Processing*, the binary system is playing an increasingly important role

Problem:

Camp Hill	Opponent
106	65
73	63
98	45
106	22
97	42
60	36
74	37
43	47
94	57
96	45
76	30
86	48
102	42
71	58
85	47
87	25
98	40
94	50
81	47
94	39
49	28
67	44
92	67
73	53
39	31
76	69
65	60
52	61

Solution:

Camp Hill Array		Opponent Array	
106		69	
106		67	
102		65	
98		63	
98		61	
97		60	
96		58	
94		57	
94	mode	53	
94		50	
92		48	
87		47	
86		47	mode
85	(83) median	47	(46) median
81		45	
76		45	
76		44	
74		42	
73		42	
73		40	
71		39	
67		37	
65		36	
60		31	
52		30	
49		28	
43		25	
+ 39		+ 22	
2,234		1,298	

Camp Hill Array

2,234 ÷ 28 = 79.78 mean

Opponent Array

1,298 ÷ 28 = 46.35 mean

Figure 21. Statistical analysis of basketball scores

in the business community. The binary system, based on two digits represented by the symbols 1 and 0, is similar in principle to the operation of a light bulb. For example, a light bulb functions in a binary mode—either on, producing light; or off, not producing light. The presence or absence of data is indicated by means of the 1 and 0 symbols. Because of its use in data processing, teachers should consider offering an introductory unit on the binary system.

Conclusion

When presenting problem solving techniques the teacher needs to present problems and numerical data which challenge and stimulate pupils. The following sequence is suggested for teaching problem solving techniques. Pupils should be taught to: (1) read and comprehend

problems; (2) recall formulas, equations and mathematical principles; (3) restate verbal problems in mathematical terms; (4) use graphics to clarify problems whenever appropriate; (5) perform the necessary calculations in the proper order and (6) state answers in correct terms.

The ability to translate verbal situations into appropriate mathematical statements will enable pupils to cope with many personal and business problems throughout their lives in an orderly and logical manner.

CHAPTER 6

EVALUATION

Just as in any other area of education, the evaluation of student progress is an integral part of the teaching process in business education classes . . . examinations can be constructed and used in ways to yield valuable information about abilities, learning difficulties, and achievements of individual students.
—Mathilde Hardaway

Evaluating pupils' fundamental skills and problem solving abilities should be considered an integral part of the instructional program. Although tests are used for arriving at grades for pupils, an analysis of their results for the planning of future instruction is of greater value.

Evaluation by Observation

Through informal evaluation, the teacher continuously should measure the effectiveness of his instruction. Many observations made in a classroom by the teacher have a bearing on a pupil's achievement. By observing pupils' reactions to instruction and by analyzing their responses to questions, the teacher can determine whether pupils comprehend the material being presented. In addition, the teacher needs to observe pupils as they work exercises and problems similar to the instructor's example on the chalkboard or at their desks.

> The alert teacher elicits student responses as he is teaching any new topic and is thus in a position to gauge the degree to which the new material is being absorbed. In addition, he provides plenty of practice and reinforcement in the development of the work as he observes signs of poor assimilation by the students. With the alert teacher, therefore, evaluation goes hand in hand with actual presentation . . . evaluation is not confined to formal testing.[1]

The teacher should counsel individual pupils concerning his observations of them so they are aware of their deficiencies and specific attitudes, work habits, computational skills or mathematical concepts needed

[1] I. David Satlow, "Problems of Teaching Business Arithmetic," *Business Education World*, 48:10, June 1968, p. 15.

for improvement. The recording of observations can be time consuming; therefore, it is essential that the teacher use a convenient check list similar to the one illustrated in Figure 22. The list of traits and work habits to be observed is suggestive and not intended to be exhaustive. Teachers are urged to add to, or delete from, this list to meet their individual situations.

BUSINESS MATHEMATICS PUPIL RATING SCALE

Name _____ Period _____

Excellent	Satis-factory	Unsatis-factory	
_____	_____	_____	Asking questions when in doubt
_____	_____	_____	Checking work for accuracy
_____	_____	_____	Completing assignments promptly
_____	_____	_____	Concentrating on explanations
_____	_____	_____	Cooperating with others
_____	_____	_____	Displaying enthusiasm and interest
_____	_____	_____	Estimating answers
_____	_____	_____	Exhibiting ingenuity and initiative
_____	_____	_____	Following directions
_____	_____	_____	Locating information
_____	_____	_____	Making optimum use of class time
_____	_____	_____	Organizing work
_____	_____	_____	Participating in class discussions
_____	_____	_____	Recording data accurately
_____	_____	_____	Working independently

Figure 22. Rating scale

Evaluation by Testing

A comprehensive, well-structured testing program should reveal strengths and weaknesses of individual pupils and in the methods and materials of instruction. A thorough analysis of test results should reveal areas for remedial teaching within the present class as well as offer suggestions for improving teaching techniques for future classes.

Teachers ought to discuss with pupils the purposes of quizzes and tests, the procedures for taking tests and the methods used in scoring them. After gaining a better understanding and insight into the testing program, pupils should be more receptive to testing situations. In addition, the teacher may find reduced tension among pupils when they comprehend the basic reasons for testing.

Formal testing in business mathematics includes the pretest, practice test, quiz, unit test, term test and final examination. Each type of test has its specific use and is discussed subsequently.

Pretest

A pretest can introduce pupils to forthcoming course work, test pupils' skills and knowledge prior to instruction and practice and serve as a motivational device. Pretest results can be used at a later date to compare with the results of a unit test, a term test or final examination. For instance, one of the instructional goals may be to have pupils increase their accuracy and speed in addition by using combinations to form ten (8 + 2, 7 + 3, 6 + 4, etc.). Here, a pretest is given to establish accuracy and speed performances for each individual. After the combination idea is presented and pupils have participated in a series of practice sessions, the same or a similar test (identical number of items of similar difficulty) is given. Scores on the two tests are compared and improvement in accuracy and in speed is stated in terms of percents.

To further stimulate the class, teachers may group pupils in teams with each team vying to achieve the greatest average increase in skill from the pretest to the unit test, term test or final examination. The teacher may choose four or five captains and then let the captains choose their teammates. This type of activity is especially applicable to the fundamental skills.

Practice Test

This test is designed for pupils to check their own progress. On duplicated practice tests, answers to exercises and problems can be included so that pupils may immediately check their responses. The answers to the questions might be listed along the right side of the test. If so, pupils can be directed to fold under the answer portion of their tests so answers will not be visible while working the problems. Tests of this type may be given for homework or during a class period.

Oral practice tests can be used when computational skills and concepts are being evaluated and when pupils' responses can be obtained easily and stated simply. The teacher can ask questions of individuals in the class or divide the class into teams.

Quiz

Quizzes should be short and their questions should be based on a specific skill or concept recently presented by the teacher and practiced by the pupils. Carefully selected quiz questions presented frequently can provide pupils with a sense of achievement in their daily

work as well as give the teacher an indication of pupils' comprehension and progress in the material being covered. Depending on its content, a quiz can be duplicated, given orally, written on the chalkboard or projected from a transparency.

If frequent assessments are to be made of a group's progress, teachers may wish to have their pupils participate in correcting quiz papers. By exchanging papers or by having the teacher collect and redistribute them, a fast and efficient method is available to pinpoint areas in which pupils are experiencing difficulty. To establish a sense of responsibility when correcting peers' quiz papers, each pupil might sign the paper he is checking. With learners participating, an item analysis can be made on the chalkboard indicating—through the use of mathematics—how *management indicators* (modes) are available. The teacher, as the *manager,* demonstrates how statistics show him where remedial work is necessary for a few pupils or for the majority of the class. Since quizzes should be administered frequently in the learning process, it is unrealistic to suggest that the teacher be required to correct all of them.

Unit Test

A unit test is based on the work of a specific unit of instruction from the textbook, text-workbook or from teacher prepared materials. The testing time usually ranges from 20 to 30 minutes. Review questions pertaining to other units of instruction should not be included in this type of test as they may affect the scores to such a degree that the unit test will not be an accurate evaluation of a pupil's progress on a specific unit of instruction. The main advantage of restricting the scope of a test to the content of a unit of instruction is its diagnostic value. Separate review tests can be given to test pupils' retention of skills and concepts previously learned.

The practice of assigning pupils to correcting quizzes should not be extended to unit tests. Some pupils, outstanding as well as low achievers, resent peers checking major tests, and rightly so. As the teacher corrects unit tests he discovers the types of errors pupils are making in arriving at incorrect solutions.

Term Test

This test should be based on a review of all material presented in a given term of instruction such as a grading period. Concentrated review should precede the test period, and pupils should be provided with a guide sheet listing the items upon which they will be tested. The term test can be compared with a pretest administered at the beginning of the term to indicate pupils' growth in computational skills and in problem solving techniques.

Final Examination

A final examination is based on all the material covered in a course. The examination should be constructed in sections covering individual skills and concepts presented during the year. In this way the teacher can ascertain which areas of instruction need additional stress in the future with implications for changes in instructional techniques and materials.

Each pupil's score on the final examination should be compared with his score on a similar test administered at the beginning of the course. If this is done, two scores should be indicated on the final examination —one indicating achievement on the final test and the other indicating the per cent of improvement over the pretest. A statistical analysis of the improvement scores—a numerical ranking from the highest to lowest score, including the mean, median and mode—would present a complete mathematical picture of the groups achievement for the year.

Preparing Test Items

The construction of test items is a process which requires careful selection and critical analysis. In preparing test items the teacher needs a thorough knowledge of business mathematics, skill in writing, a knowledge of test construction and an awareness of pupils' achievement and abilities. Specifically, the teacher needs to consider the following points.

First, identify each test item with the specific objective it is intended to measure so pupils' responses will indicate their understanding of that objective. Therefore, the writing or selection of test items should begin with a reference to the objectives for that lesson, unit or term.

Second, include items of varying difficulty to measure the entire range of abilities within a given class. There should be questions easy enough for all pupils to answer and difficult questions which would make perfect test scores improbable. The teacher should arrange the items in order of their difficulty, the easier questions first and the more difficult questions following. Experiencing success with the easier questions will help pupils acquire confidence which may give them impetus and enthusiasm to solve the more difficult exercises and problems. By having the questions in a test arranged in this manner, the teacher also can pinpoint the level at which the class and individuals need remedial instruction, and plan his teaching on this basis.

Third, state test items in language that is concise and comprehensible by all pupils. Except for those questions which are designed for pupils to distinguish between essential and nonessential data, test items should be void of irrelevant data and ambiguous language. As a check on the

readability of test items, another business teacher might take the test and offer suggestions for rewriting items which may not be clearly stated.

Fourth, use various forms of test items. These forms include objective items (completion, multiple choice, true or false, yes and no and matching), mathematical items (composed of figures and symbols only) and verbal problems.

Fifth, write the test items daily on index cards and file them according to objectives or concepts for future use. After each objective, the test items should be arranged according to their difficulty. This method enables teachers to add, delete and revise individual test items with ease. When sufficient items have been accumulated under each objective or concept, teachers can assemble a test when needed.

Sixth, provide adequate space on test papers or on answer sheets for performing computations and for writing answers. Some pupils' normal handwriting is large, and they need sufficient space to work the exercise or problems. If pupils are forced to write in cramped quarters, they may have difficulty with number alignment, causing errors in computation. For scoring purposes, there are advantages in having a line or space designated for the answer to each test item.

Seventh, evaluate the test in its entirety as well as the individual items. After scoring a test, make an item analysis by counting the number of pupils who made incorrect responses to each item. Test items which have a high incidence of incorrect responses should be analyzed by responding to the following questions:

1. Was the test item clearly stated?
2. Was the difficulty level of the item out of the range of most pupils within the class?
3. Should more time be devoted to teaching this concept?
4. Should a different approach or instructional technique be used to reteach this concept?
5. Are visual aids or other teaching materials available for use in reteaching this concept?
6. Were the pupils aware of the importance of this concept?

Item analysis can aid teachers in planning instruction and in building an evaluation instrument which will serve as a more effective teaching tool in the future.

Sample Test Items

Various types of test items are available for use in testing pupils' knowledge and skills. In addition to commonly used test items composed of figures and symbols only, teachers may want to consider making use of other forms. The following items are selected examples for teachers' perusal.

Completion

1. A decrease in the value of property is referred to as *depreciation.*
2. The statement, "There are 15 boys out of a total of 25 pupils in the business mathematics class" may be expressed in fractional terms as *3/5.*

Multiple Choice

1. To find the perimeter of a circle, you multiply 3.1416 by the (a) circumference, (b) *diameter,* (c) radius.
2. The product of .37 1/2 × 240 is (a) 30, (b) 60, (c) *90.*

True or False

1. The formula for finding interest is as follows: Interest = Principal × Rate × Time. *True.*
2. If a bag contains 5 white marbles and 10 red marbles, the probability of picking a red marble is 1.3. *False.*

Yes and No

1. Are personal property taxes assessed on real estate? *No.*
2. Is the rate 2.5% equivalent to 25/1000? *Yes.*

Matching

b 1. 1/4	(a)	.875
d 2. 2/5	(b)	.25
a 3. 7/8	(c)	.90
c 4. 9/10	(d)	.40
	(e)	.625

Essay

1. Explain the meaning of the fraction 3/5.
   Three parts have been selected from an amount, quantity or object having five equal parts.

2. Translate the following mathematical statement into a verbal statement relating it to the equal fractions principle: \$12.80 = 4% × \$320

\$12.80 compares to \$320 as 4% compares to 100%

Graphics

1. Using graphics, illustrate the similarity between the fraction 1/5 and the per cent 20.

1/5	1/5	1/5	1/5	1/5	= 5/5 or 1 (whole)
20%	20%	20%	20%	20%	= 100% or 1 (whole)

2. Using graphics, illustrate the improper fraction 5/4.

1/4	1/4	1/4	= 5/4 or 1 1/4
1/4	1/4		

The graphics used in the solutions are suggestive, and pupils undoubtedly will develop different illustrations.

Conclusion

Evaluation is an essential aspect of instruction. If pupils are to achieve their optimum potential, effective evalaution should indicate whether the teacher's instruction is being transferred into pupils' learning. To a degree evaluation also can be used as a stimulant, demonstrating to pupils that they are capable of solving business mathematics problems. A teacher needs to use every effective evaluative technique available to arrive at the chief goal of instruction; that is, to improve the progress of each pupil.

APPENDIX A
PLANNING FOR INDIVIDUALIZED INSTRUCTION

During the past decade there has been much discussion on individualizing instruction. A valid question on structuring a course geared to individualization is, "Where do I begin?" The initial step is to decide specifically what is to be taught. A basic outline of the topics a teacher wishes to include in the course should be made. The following material describes the sequence which should be followed in preparing each unit of instruction.

Behavioral Objectives

A list of behavioral objectives, which indicate what the teacher expects his learners to demonstrate upon successfully completing the unit, needs to be prepared. The objectives should be written clearly and, according to Mager,[1] include the following:

1. The statement should identify what the learner will be doing when he is demonstrating that he has reached the objective.
2. The statement should describe the important conditions (givens or restrictions, or both) under which the learner will be expected to demonstrate his competence.
3. The statement should indicate how the learner will be evaluated. It should describe at least the lower limit of acceptable performance.

Identifying Terminal Behavior

When identifying terminal behavior, phrases such as *to compare, to compute, to demonstrate, to find, to identify, to illustrate, to recite, to solve,* or *to write* should be used. Avoid nebulous phrases such as *to know* and *to understand,* unless the objective specifically indicates how it is to be determined that the learner knows or understands.

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[1] Robert F. Mager, *Preparing Instructional Objectives,* (Palo Alto, California: Fearon Publishers, 1962), p. 12.

Describing Important Conditions

The conditions under which the behavior will be expected to occur refers to the difficulty of the problems, what type of information will be given, whether the learner may use his textbook or other reference source and if the learner is required to do the problem mentally or by using pencil and paper.

Indicating Acceptable Performance

The criteria of acceptable performance may include a specific time limit for a given problem or group of problems, a minimum number of correct responses out of a group or a percentage of correct responses.

Sample Behavioral Objectives

Including the previous three points in each objective, whenever possible, increases the communicative value of the statement. The following behavioral objectives are presented as examples.

1. Given the common fractions 1/8, 3/8, 5/8, 7/8, 1/4, 3/4 and 1/2, the learner can recite their decimal equivalents correctly within 15 seconds.
2. Given problems containing fractional parts of pounds (such as 2/3 lb.) to be multiplied by mixed fractions representing cents (such as 15 1/4 cents), a pupil can—with pencil and paper—find the correct selling prices in 80 per cent of the problems.
3. Provided with the financial section of a newspaper and a list of 10 selected stocks, a pupil can list the opening and closing quotations in dollars and cents without referring to a table of decimal equivalents.
4. Given 10 multiplication problems involving mixed numbers, such as 24 1/3 × 5 2/4, the learner, using pencil and paper, can arrive at correct solutions in at least 8 of the problems.
5. A learner can demonstrate his understanding of fractions by writing a definition for a fraction, using the terms *numerator* and *denominator* correctly.
6. The learner can illustrate the fraction 5/6 by drawing a pictorial pie graph.

Pretest

A pretest should be prepared for each unit of instruction. The test items should be based on the objectives established. Results of the test will then indicate an individual's strengths and weaknesses as they

relate to those objectives. If a pupil demonstrates a thorough mastery of all the material, he can be assigned an enrichment exercise of a more difficult nature not required of all pupils.

Prescribed Instruction

On the basis of the pretest, the teacher prescribes a series of instructional guides for pupils to study and work sheets for pupils to use. The instructional guides may include duplicated materials prepared by the teacher, transparencies, tape recordings and specific pages in one or more textbooks. The work sheets contain exercises to develop competence in meeting those objectives in which the learner lacks a given knowledge or skill. In some schools, most of the instructional guides and work sheets are duplicated and stored in specially built racks which are placed along one of the classroom walls.

Post Test

After the learner successfully completes his prescribed instructional guides and work sheets, he is given a post test (unit test) to determine whether he can demonstrate in terms of correct responses his knowledge and skills according to the objectives for that unit. The test items, based upon the behavioral objectives established for that unit, should be developed in sufficient number to adequately test a pupil's competence. If the test indicates the pupil is not satisfactorily performing one or more of the objectives, additional study-work sheets are prescribed after which a second post test is administered.

Considerations

Writing behavioral objectives, developing instructional guides, preparing work sheets and constructing pretests and post tests are time consuming projects. Developing units of individualized instruction is the type of activity which should be engaged in during the summer months. Some school districts establish workshops for this purpose.

Individualized instruction may be the answer to some of today's educational problems when a wide range of abilities is present within a single class or in those schools where absenteeism may be a chronic problem. It is not, however, a panacea for all educational shortcomings. Some teachers using this approach have observed that the highly motivated pupils seem to benefit most from this type of instruction because they are geared toward working on their own, they enjoy the independence afforded by this approach and are not held back by slower learners. On the other hand, some of the slower, less motivated pupils have been observed to slow down to a greater degree in a class conducted

on an individualized basis and need to be constantly prodded. These observations depend upon the variables present, including the individual pupil, his attitude and ability; the instructor, his attitude and resourcefulness; and the individualized instructional materials, their quality and quantity.

Another facet of the individualized approach of which teachers should be aware is the necessity for keeping complete and accurate records. Each pupil's record would include the scores he made on each section of the diagnostic pretest for each unit, the instructional materials the teacher prescribes for each unit, the pupil's progress in completing study guides and work sheets and his success or failure in the post achievement test.

Before developing an entire course based on individualized instruction, the teacher ought to develop one or several units to cover a single grading period. After using these materials for one or more groups, the teacher will realize some of the shortcomings in the materials he has developed. He is then in a much better position to revise his original materials and work toward developing materials for an entire course.

APPENDIX B
PUBLICATIONS

Many publications related either directly or indirectly to business mathematics are available. A partial listing includes those found in this appendix.

Books

Some books of a general nature that would be of interest to business mathematics teachers include those indicated below.

Butler, Charles H., F. Lynwood Wren and J. Houston Banks, *The Teaching of Secondary Mathematics.* New York: McGraw-Hill Book Company, Inc., 1970. 597 pp.

Hardaway, Mathilde, *Testing and Evaluation in Business Education.* Cincinnati: South-Western Publishing Company, 1966. 487 pp.

Johnson, Donovan A. and Gerald R. Rising, *Guidelines for Teaching Mathematics.* Belmont, California: Wadsworth Publishing Company, Inc., 1967. 446 pp.

Lindeman, Richard H., *Educational Measurement.* Glenview, Illinois: Scott, Foresman and Company, 1967. 173 pp.

Mager, Robert F., *Preparing Instructional Objectives.* Palo Alto, California: Fearson Publishers, 1962. 61 pp.

Willoughby, Stephen S., *Contemporary Teaching of Secondary School Mathematics.* New York: John Wiley & Sons, Inc., 1967. 430 pp.

Free and Inexpensive Materials

Reference in this bulletin is made to a number of the following items. Because a large quantity of this type of material is available, the following list should not be considered as exhaustive.

Field Enterprises Educational Corporation, Box 3565, Merchandise Mart Plaza, Chicago, Illinois 60654

The following pamphlet provides an introduction to set theory and to numeration systems. Many illustrations are included, some of which can be used in classroom presentations.

New Math, Reprint No. SA-2286

Institute of Life Insurance, Educational Division, 277 Park Avenue, New York, New York 10017

In addition to the following publications other booklets are available. Write for a current catalog.

Handbook of Life Insurance
Policies for Protection
Sets, Probability and Statistics

Insurance Information Institute, Educational Division, 110 William Street, New York, New York 10038

This 32-page booklet offers an introduction to the theory of probability through programmed instruction.

Chances Are

National Council of Teachers of Mathematics, Inc., 1201 Sixteenth Street, N.W., Washington, D.C. 20036

The following paperback publications, which present expositions of basic topics in mathematics, may be purchased from the NCTM.

Booklet 1. *Sets*
Booklet 2. *The Whole Numbers*
Booklet 3. *Numeration Systems for the Whole Numbers*
Booklet 4. *Algorithms for Operations with Whole Numbers*
Booklet 5. *Numbers and Their Factors*
Booklet 6. *The Rational Numbers*
Booklet 7. *Numeration Systems for the Rational Numbers*
Booklet 8. *Number Sentences*
Booklet 9. *The System of Integers*
Booklet 10. *The System of Rational Numbers*
Booklet 11. *The System of Real Numbers*
Booklet 12. *Logic*
Booklet 13. *Graphs, Relations, and Functions*
Booklet 14. *Informal Geometry*
Booklet 15. *Measurement*
Booklet 16. *Collecting, Organizing, and Interpreting Data*
Booklet 17. *Hints for Problem Solving*
Booklet 18. *Symmetry, Congruence, and Similarity*

New York Stock Exchange, 11 Wall Street, New York, New York 10005

A 48-page textbooklet which includes types of business organizations, stocks and bonds, sources of information on investments and capital in the economy is offered.

You and the Investment World

SCM Corporation, 299 Park Avenue, New York, New York 10017

This publication presents a brief history of figuring from fingers to calculators.

From Og . . . to Googol

Victor Educational Services Institute, 3900 North Rockwell Street, Chicago, Illinois 60618

This 15-page booklet presents a review of basic mathematical terms and applications and is available free in reasonable quantities for classroom use.

Victor Refresher

APPENDIX C
UNDERSTANDING NEGATIVE NUMBERS

There is a definite need for developing the understanding of negative numbers. While understanding negative numbers is no more important than many other business mathematics applications, its absence from business education publications suggests that it has not been considered in the past and should be included in the development of future courses of study.

While *credit balance* is a negative number used frequently in the business world, the term is not completely understood by all who use it. To develop understanding of the credit balance concept, pupils need to know how and why negative balances occur in business transactions. Some of the situations which involve negative numbers and may result in credit balances include returning merchandise previously paid for, overpaying an invoice, discounts applied to accounts after payment of gross amount is received, overdrawing an expense allowance, cancelling a check after it has been recorded in the bookkeeping records and a previously deposited check returned by the bank due to insufficient funds in the drawer's account.

A number line can be used to illustrate what causes a negative number to develop and may help pupils gain insight. For instance, a manufacturer's representative is alloted $150 per month for expenses; however, during the month of February, he spends $165. When the bookkeeper subtracts $165 from $150, the result is $15 CR (a negative number) as illustrated in Figure 23.

An adding machine tape would list a credit balance in the manner illustrated in Figure 24.

There are various ways of indicating negative numbers on financial records as shown in Figure 25.

Another way to indicate negative numbers is by using red ink for pen written figures and a red machine ribbon for printed figures. Using this system, all positive numbers would be indicated by black figures.

Exercises for developing skill in working with negative numbers can be presented in novel and interesting ways. Under the teacher's direction, pupils may prepare 5 × 3 cards, one set containing one- and two-

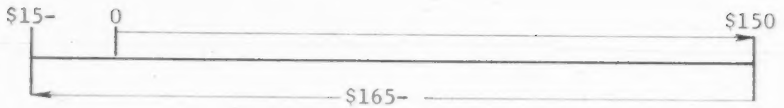

Figure 23. Number line

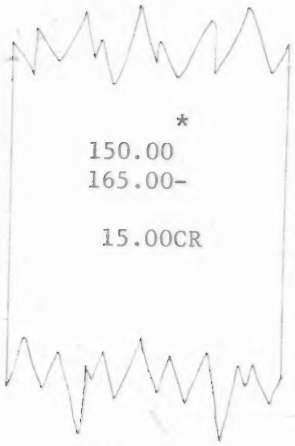

Figure 24. Credit balance

11.25-

11.25CR

11.25C

(11.25)

Figure 25. Negative number

digit numbers written in black ink and the other set having numbers written in red ink. The cards remain in separate packs according to their color. Each pack of cards is shuffled separately and placed on a table with their numbers facing upwards. As a pupil looks at the two numbers, he performs the necessary calculation mentally. If the red number is more than the black number, a minus number results; if it is less than the black number, a positive number results. The top card in each pile is removed and the next two cards are compared. When all the cards have been used, they may be reshuffled and used again.

Two examples are given to illustrate the use of negative numbers in an office situation.

Example 1. The members of a Future Business Leaders of America chapter sell imprinted stationery, note paper and ballpoint pens as one of their projects. They have an account with the Modern Printing Company and purchased the following merchandise during the month of October: 60 boxes of note paper, $30; 150 ballpoint pens, $67.50; 75 boxes of stationery, $56.25; and 40 additional boxes of note paper, $20. On November 10, the FBLA chapter receives an invoice for the purchased items in the amount of $173.75, which the club's treasurer immediately pays by check. After paying for the merchandise, one of the pupils found that the 25 pens in one box are defective. Upon returning the defective pens to Modern Printing Company, the club receives a credit memorandum in the amount of $11.25, along with an explanation that it is not economical for the company to imprint only 25 pens. Modern Printing Company's record of this FBLA account will resemble that shown in Figure 26.

Pupils should understand that the last figure indicated in the balance column is usually the amount the customer owes the company; however, when the last figure is shown with a CR symbol, this indicates that the company owes the customer money or merchandise of that amount. In the illustration presented in Figure 26, the Modern Printing Company owes the FBLA chapter $11.25 or merchandise valued at that amount.

Example 2. Another use of the negative number is in business recordkeeping when similar transactions need to be kept in the same column of an account. To accurately record deposits and payments, all deposits and transactions pertaining to deposits should be recorded on the debit side. All payments and transactions pertaining to payments should be recorded on the credit side. For illustrative purposes, the following transactions are recorded in "T" account form in Figure 27.

MODERN PRINTING COMPANY

Monthly Statement

TO: *Future Business Leaders of America*
University High School
Localtown, Pa. 12345

Account No. 24 56 38

Date	Purchases	Payments & Returns	Balance
Oct 5	*30.00*		*30.00*
Oct 12	*67.50*		*97.50*
Oct 19	*56.25*		*153.75*
Oct 25	20.00		*173.75*
Nov 15		*173.75*	*—0—*
Nov 18		*11.25*	*11.25 CR*

Figure 26. Monthly Statement With Credit Balance

C A S H

(A) 250	(B) 25
(D) 75	(C) 42
(F) 15 -	(E) 36
	(G) 18
	(H) 42 -
310[a]	79[b]

231[c]

"a" represents the total amount of deposits—$310

"b" represents the total amount of checks written—$79

"c" indicates the new balance—$231

Figure 27. "T" Account

Transactions:

A. Deposit of $250

B. Check written for $25

C. Check written for $42

D. Deposit of $75

E. Check written for $36

F. Check for $15 returned by bank
This check was marked insufficient funds. It was part of the total deposit of $250 and should reduce the total amount of deposits by $15.

G. Check written for $18

H. Cancelled check previously written for $42
The check was lost by the payee. The original entry in the records was a credit to cash; however, if the cancellation entry should be entered as a debit to cash, it would distort the true story as to the total deposits and therefore should be entered on the credit side as a minus figure as shown.